Sea level variations during snowball Earth formation and evolution: 2. The influence of Earth's rotation

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[1] Preliminary analyses are described of the influence of snowball Earth formation on the rotational state of the Earth as well as its feedback onto relative sea level. We demonstrate that a sufficiently large excess ellipticity of the Earth as might be expected to arise due to the mantle convection process acts to stabilize the rotational axis significantly so that the associated relative sea level change would be negligible. If no such excess ellipticity were characteristic of Neoproterozoic time, then increasing the thickness of the elastic lithosphere significantly promotes true polar wander (TPW) and the associated relative sea level change. On the contrary, increasing the viscosity of the lower mantle has an equally significant but opposite effect. TPW due to ice sheets formation for the 720 Ma and 570 Ma continental configurations (approximate Marinoan) can reach more than 5° and 10° in 10 Myr for viscosity model VM5a, and the associated maximum relative sea level changes at this time reach 26 m and 49 m, respectively. However, if a 1°/Myr TPW due to the action of the mantle convection process is assumed to be superimposed, then these values increase to 70 m and 101 m respectively. Compared to the analyses in which rotational influence is entirely neglected, the probability density distribution of freeboard values obtained here is almost the same except that the tails of the distribution are broadened, making it more difficult to accurately infer continental ice volume during snowball Earth events from observed freeboard changes.

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1. Introduction

[2] A preliminary analysis of sea level variations during snowball Earth formation was described in *Liu and Peltier* [2013], in which the primary component of Neoproterozoic sea level change due to ice sheet formation and the associated isostatic adjustment processes were analyzed, while the additional component due to both rotational feedback and the secular variation of the Earth's rotational state due to mantle convection and tidal friction was ignored. This separation of the work into two parts was suggested by the fact that the influence of rotational feedback on the long time scales of interest is complicated by the unidirectional true polar wander (TPW) that could be induced by the ice sheet loading of the continents and due to the underlying action of the mantle convection process itself. In considering the impact of variations in Earth's rotational state on the sea level changes expected during a snowball Earth glaciation event, several additional issues must be taken into account that are significantly less relevant when such variations are neglected. These include, first, the fact that ice sheet loading–induced TPW is sensitive to the radial structure of mantle viscosity [e.g., *Peltier*, 1998], an influence that was argued to be almost irrelevant in *Liu and Peltier* [2013]. Second, the relative sea level change due to this surface load–induced TPW may not reach a steady state on the time scales of interest for a Neoproterozoic glacial cycle. Third, mantle convection itself may be responsible for simultaneously inducing long-term TPW that would be superimposed on that generated by the surface ice sheet loading event. Fourth, the possible existence of an excess ellipticity of the Earth such as is characteristic of the present planetary shape (e.g., see *Peltier and Luthcke*, [2009] and *Peltier et al.*, [2012] for recent discussions) may have a significant effect on the TPW induced by ice loading. Finally, although it has been demonstrated previously [*Mound et al.*, 1999] that tidal deceleration of the rate of planetary rotation should play a negligible role in long-term relative sea level change if a constant deceleration characteristic of the past few hundred million years is assumed, it will be interesting to understand whether this influence could become more significant if the deceleration were larger during Neoproterozoic time. There is also an additional and perhaps more fundamental complication that arises associated with application of the theory developed as the basis on which to analyze the influence of rotational feedback [e.g., *Sabadini and*

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Peltier, 1981; *Peltier*, 1982; *Sabadini et al*., 1982; *Wu and Peltier*, 1984] for application to the late Quaternary ice ages to the Neoproterozoic problem. Here the issue is that this theory may become invalid on the long time scales associated with the Neoproterozoic glaciations as the existing methodology is based on the application of first-order perturbation theory.

[3] During the late Quaternary quasiperiodic ice age cycle, it is well understood that the variations of the total moment of inertia tensor of the planet are describable in terms of linear perturbation theory. This is both because the characteristic time scale of such variations is fast compared to the time scale of the mantle convection process and, more importantly, because the variations themselves are relatively small. However, the time scale associated with snowball Earth glaciation events is probably long (4–30 Myr; e.g., *Hoffman et al*. [1998] and *Macdonald et al*. [2010] constrained the duration of Sturtian glaciation to be at least 5 Myr), and the associated perturbations to the moment of inertia tensor are much larger than those associated with late Quaternary glacial cycles. The fact that the time scale of the Neoproterozoic glaciations is such that significant rearrangement of the continents could be thereby induced, either through the direct influence of forced plate tectonic processes or by convection-induced true polar wander, implies that a clear separation of glaciation- and convection-related influences will, in general, be impossible to achieve. Over time scales of millions or tens of millions of years, the Earth's rotational axis might be expected to migrate by many degrees relative to the mean mantle through the action of true polar wander (TPW) under the action of such a large and persistent perturbation, sufficient to render the assumptions that underlie the traditional theory invalid. In this theory, it has always been assumed that the angular velocity (both direction and speed) changes very little during the ice ages [e.g., *Sabadini and Peltier*, 1981; *Peltier*, 1982; *Sabadini et al*., 1982; *Wu and Peltier*, 1984].

[4] To determine the influence of Earth's rotation on Neoproterozoic sea level change, the evolution of the rotational velocity of the planet needs to be known, and this is determined by the time dependence of the moment of inertia tensor. This tensor may be thought of as consisting of three parts [e.g., *Cambiotti et al*., 2010, 2011]: that of a nonrotating spherically symmetric Earth; that due to the rotation of the Earth, which causes the Earth to assume the form of a flattened ellipsoid of revolution; and, finally, that due to the mass redistribution within and on the surface of the Earth due to continuously evolving mantle convection and to ice age–related redistributions of mass between the oceans and the continents. Assuming the Neoproterozoic Earth to be similar to present day insofar as its time mean state is concerned, the first and second contributions to the moment of inertia can be estimated given the internal density structure determined seismologically and an assumed known rate of axial rotation. The third contribution is less completely understood due to both inadequately constrained mantle convection [*Mitrovica et al*., 2005; *Peltier and Luthcke*, 2009; *Cambiotti et al*., 2010] and uncertainties in the state of Neoproterozoic glaciation (e.g., "hard" or "soft" snowball Earth, see below).

[5] If the rotational axis of the Earth were to be shifted by more than approximately 10°, the direct effect of increased solar insolation on the ice sheets (since they will be wandering equatorward) would potentially be significant, especially if the glaciated state were a "soft" rather than a "hard" snowball. An even more significant influence could arise from the change in the concentration of greenhouse gases [e.g., *Le Hir et al*., 2008], which could dramatically increase on the time scale of millions of years (i.e., that of snowball Earth glaciation), thereby acting to further decrease the volume of land ice during the migration of the glaciated continents toward the equator. Eventually (after 4–30 Myr), due to the action of these two feedbacks, perhaps aided by other processes [e.g., *Abbot and Halevy*, 2010; *Abbot and Pierrehumbert*, 2010; *Le Hir et al*., 2010], even a hard snowball Earth could be deglaciated. Additional problems arise due to the fact that continents evolve with time and perhaps significantly even on a time scale of a few tens of millions of years (since the upper bound on the duration of a snowball Earth event remains relatively unconstrained [*Hoffman and Li*, 2009]), especially during the Neoproterozoic era [e.g., *Li et al*., 2008]. In the present paper, our intention is to consider how snowball Earth formation affects sea level and Earth's rotation for two fixed realistic continental and ice sheet configurations [*Liu and Peltier*, 2011].

[6] For the reasons previously discussed, we do not think that it is reasonable to attempt to compute Earth's rotational and sea level response to Neoproterozoic glaciation for time scales longer than a few million years if the TPW is fast or tens of millions of years if it is slow. In either case, it will be irrelevant as to whether the theory used to calculate the rotational response is linear (traditional) or nonlinear [e.g., *Ricard et al*., 1993]. Our analyses here are rather intended to provide a basis on which the feedback of varying solar insolation and sea level change on planetary climate and related continental ice cover may be further addressed in the future.

[7] The strategy that we will adopt to attack this problem is one in which we will first analyze the nature of relative sea level change to be expected under prescribed perturbations to the rotational velocity. We will consider perturbations as large as a 90° shift in direction and a 0.45% change in the rate of rotation. In this simple case, the technique employed will be the same as that employed in *Mound et al*. [1999], but here we will analyze the results more systematically and in the specific context of snowball Earth formation. Sensitivity tests will be performed for a variety of different rates of TPW and eight different viscosity models with more focus on the lower mantle viscosity since the rotational variables are most sensitive to variations in viscosity in the deepest part of the lower mantle, as evidenced by the appropriate Frechet derivatives [e.g., *Peltier*, 1998]. In the second stage of analysis, we will employ the traditional linear theory [e.g., *Sabadini and Peltier*, 1981; *Peltier*, 1982; *Sabadini et al*., 1982; *Wu and Peltier*, 1984] to calculate the amount of TPW of the Earth that would occur due to ice sheet formation during the first 10 Myr of a Neoproterozoic glaciation event. As will be demonstrated in the results section, the TPW is fast only during the formation stage and slows rapidly after the glaciation process is completed. Subsequent to this, TPW continues at a slow speed (usually $<1°$/Myr) for a considerable length of time until the snowball Earth is melted, and the associated relative sea level changes (at locations where these changes are most sensitive to perturbations to the rotational state of the Earth) are moderate and almost linear with time. The traditional theory is therefore sufficient to allow us to calculate the variation of Earth's rotation during the period that is of greatest interest, namely, the formation stage. The

results from the first step of analysis will prove to provide excellent guidance as to how relative sea level might change if the snowball Earth event were to persist for a time scale longer than 10 Myr.

[8] The first-order perturbation theory–based analysis is well suited for calculating the influence of the shorter length of day (LOD) during Neoproterozoic time on relative sea level. Moreover, the linear theory of *Peltier* [1982] and *Wu and Peltier* [1984] has been modified slightly [*Mitrovica et al.*, 2005; *Peltier and Luthcke*, 2009] to enable consideration of the possible influence of excess ellipticity of planetary shape (i.e., the Earth is observed at present to be very slightly more flattened than predicted by steady state theory for a spherically symmetric Maxwell Earth model [*Peltier*, 1974, 1976; *Peltier and Luthcke*, 2009]) on the rotational response to the late Quaternary glacial cycles. Following the methodology of *Peltier and Luthcke* [2009], we will also consider the possible influence of excess ellipticity on the rotation of the Earth on the long time scale associated with Neoproterozoic snowball Earth formation and evolution, although the existence and cause of such excess ellipticity (especially during the Neoproterozoic) are both subjects of current debate.

[9] In the analyses to follow, we will first briefly summarize in section 2 the method employed in the traditional theory [*Sabadini and Peltier*, 1981; *Peltier*, 1982; *Sabadini et al.*, 1982; *Wu and Peltier*, 1984] to calculate the expected relative sea level change due to rotational feedback. In this process, we will also address the question as to how the influence of a shorter Neoproterozoic LOD and possibly excess ellipticity of the Earth can be considered. Following this brief review of theory, we describe in section 3 how appropriate experiments may be designed to study the relative sea level change due to a prescribed change of rotational state. This is followed immediately by the presentation and discussion of the results of these experiments. In section 4, we next establish the validity of the software to be employed to calculate the influence on the rotation of the Earth due to the growth of a large ice sheet by employing an ideal circular continent whose initial location is taken to be a function of latitude. The influence of the loading history associated with the ice sheets that form during a realistic snowball Earth event on the planet's rotational state is then considered, with the initial focus being on the influence of a shorter LOD and excess ellipticity. Finally, the influence on relative sea level change due to the TPW due to a combination of both surface ice loading and internal mantle convection is discussed. Our purpose in this section is to estimate an upper bound on the influence of a variation in the rotational state on relative sea level change and its temporal evolution, thereby providing guidance as to how the observed relative sea level change might be used to infer continental ice volume during a Neoproterozoic glaciation event.

2. Theoretical Background and Model Design

[10] The same radial structure of the Earth will be employed as assumed in *Liu and Peltier* [2013], i.e., the density and elastic properties are those of the preliminary reference Earth model [*Dziewonski and Anderson*, 1981], and the radial viscosity structure is chosen from a set of eight different models, including the VM5a model [*Peltier and Drummond*, 2008]. The other seven viscosity models are as follows: VM5b, for which the only difference from VM5a is that the viscosity of the upper mantle is reduced from 0.5×10^{21} to 0.25×10^{21} Pa s; VM5aL90 and VM5aL120, which are the same as VM5a but the thickness of the elastic lithosphere (treated as a very high viscosity layer) is increased from 60 km to 90 and 120 km, respectively; lm4um1L90, lm10um1L90, lm30um1L90, and lm100um1L90, which are all simple three-layer viscosity models, in which the thickness of the elastic lithosphere is 90 km, the viscosity of the upper mantle is 1×10^{21} Pa s, but the viscosity of the lower mantle is varied through the sequence of values of 4×10^{21}, 10×10^{21}, 30×10^{21}, and 100×10^{21} Pa s, respectively. The viscosity of the elastic lithosphere for all models is assumed to be effectively infinite. The method employed here to calculate the influence of rotational feedback on sea level is the same as that in *Peltier and Luthcke* [2009], with refinements recently discussed in *Peltier et al.* [2012].

[11] To help explain how a changing LOD and additional factors will be represented herein, the necessary theoretical results from *Peltier and Luthcke* [2009] will be briefly recapitulated. To begin, we may restate here the sea level equation in *Liu and Peltier* [2013, equation (2)] as

$$\begin{aligned}\delta S(\lambda,\theta,t) = {} & C(\lambda,\theta,t)[\delta G^L(\lambda,\theta,t) - \delta R^L(\lambda,\theta,t) + \delta G^T(\lambda,\theta,t) \\ & -\delta R^T(\lambda,\theta,t)] + \delta C(\lambda,\theta,t)[G^L(\lambda,\theta,t) - R^L(\lambda,\theta,t) \\ & + G^T(\lambda,\theta,t) - R^T(\lambda,\theta,t)]\end{aligned} \tag{1}$$

where $\delta G^T(\lambda,\theta,t)$ and $\delta R^T(\lambda,\theta,t)$ are the geoid perturbation and the radial displacement perturbation of the surface of the solid Earth due to rotational feedback, respectively, influences that were ignored in *Liu and Peltier* [2013]. They can be calculated by evaluating the following convolution integrals:

$$\delta G^T(\lambda,\theta,t) = \delta\Psi(\lambda',\theta',t) * G^T_\varphi(\varphi,t) \tag{2a}$$

$$\delta R^T(\lambda,\theta,t) = \delta\Psi(\lambda',\theta',t) * G^T_R(\varphi,t) \tag{2b}$$

respectively, where $\delta\Psi(\lambda',\theta',t)$ is the perturbation to the centrifugal potential at location (λ',θ'), with λ' and θ' as the longitude and the latitude, respectively. $G^T_\varphi(\varphi,t) = \frac{5}{4\pi}\cdot\frac{1}{g}\left[\delta(t) + k^T_2(t)\right] P_2(\varphi)$ and $G^T_R(\varphi,t) = \frac{5}{4\pi}\cdot\frac{h^T_2(t)}{g} P_2(\varphi)$ are degree 2 tidal Green functions for geoid perturbation and radial displacement of the solid Earth surface, respectively, where $k^T_2(t)$ and $h^T_2(t)$ are the viscoelastic tidal Love numbers that describe the response of the Earth to a general potential forcing [see, e.g., *Peltier*, 2007, equations (15a) and (15b)]. $P_2(\varphi)$ is the second-degree Legendre polynomial, where φ is the angular distance between the location of the potential perturbation (λ',θ') and the location (λ,θ) where the response is evaluated. The operator "*" in equation (2) represents convolution in both space and time; however, the renormalization factor $\frac{5}{4\pi}$ in $G^T_\varphi(\varphi,t)$ and $G^T_R(\varphi,t)$ eliminates the spatial part of the convolution operation [*Peltier et al.*, 2012].

[12] Our focus here will be on $\delta\Psi(\lambda',\theta',t)$, which is a function of the rotational state of the Earth and may be written in the form [*Dahlen*, 1976]

$$\delta\Psi(\lambda',\theta',t) = \delta\Psi_{00}(t)Y_{00}(\lambda',\theta',t) + \sum_{n=-2}^{+2}\delta\Psi_{2n}(t)Y_{2n}(\lambda',\theta',t) \tag{3}$$

with

$$\delta\Psi_{00} = \sqrt{4\pi}\frac{\Omega^2 a^2}{3}\left[m^2 + 2m_3\right] \tag{4a}$$

$$\delta\Psi_{20} = \sqrt{4\pi}\frac{\Omega^2 a^2}{6\sqrt{5}}\left(m_1^2 + m_2^2 - 2m_3^2 - 4m_3\right) \tag{4b}$$

$$\delta\Psi_{21} = \sqrt{4\pi}\frac{\Omega^2 a^2}{\sqrt{30}}\left[(1+m_3)m_1 - i(1+m_3)m_2\right] \tag{4c}$$

$$\delta\Psi_{22} = \sqrt{4\pi}\frac{\Omega^2 a^2}{\sqrt{120}}\left(m_2^2 - m_1^2 + i2m_1m_2\right) \tag{4d}$$

$$\delta\Psi_{2,-n} = (-1)^n \delta\Psi_{2n}^* \tag{4e}$$

where Y_{00} and Y_{2n} are orthonormalized spherical harmonic functions of degrees 0 and 2, and Ω and a are the angular velocity and the mean radius of the Earth, respectively. Superscript "*" in the above equation denotes the complex conjugate of the basis function $m^2 = m_1^2 + m_2^2 + m_3^2$, where m_1, m_2, and m_3 are defined so that the components of the new angular velocity in the reference frame are

$$\omega_i(t) = \Omega(\delta_{i3} + m_i(t)), \quad i = 1, 2, 3 \tag{5}$$

[13] Compared to *Peltier* [2007, equation (13)], our equation (3) here retains all terms of spherical harmonic order 2 ($\delta\Psi_{22}$ and $\delta\Psi_{2,-2}$) so that in equation (4), all the second-order terms of m_i are also retained. This is because the TPW angle will be large in several of the simulations to be discussed herein, in which case these terms are no longer small. They may be neglected [e.g., *Peltier*, 2007] in simulating the TPW associated with the Pleistocene ice ages due to the small TPW angle achieved during a typical glaciation cycle, for which the degree 2 and order 1 terms ($\delta\Psi_{21}$ and $\delta\Psi_{2,-1}$) dominate [e.g., *Sabadini et al.*, 1990].

[14] Equations (1)–(5) are sufficient for calculating the relative sea level change due to a prescribed change of the rotational state of the Earth [e.g., *Sabadini et al.*, 1990], although in our calculations, the loading change on the ocean floor predicted by the sea level equation constrained redistribution of ocean water is also considered as in *Mound et al.* [1999]. In order to calculate the change in planetary rotation due to ice sheet growth, the change of the moment of inertia tensor in an appropriate reference frame must be tracked. The origin of this reference frame is the center of mass of the Earth, with the third axis aligned with the spin axis prior to perturbation, and the first and second axes are in the equatorial plane and are orthogonal. In the description and derivations to follow, it will be assumed that $m_i << 1$ as the evolution equations (below) employed for the moment of inertia changes are only first order accurate. The individual components of the new rotation vector in equation (5) are determined by solving the appropriate Euler equation for a system subjected to no external torque, namely,

$$\frac{d}{dt}J_{ij}\omega_j + \epsilon_{ijk}\omega_j J_{kl}\omega_l = 0 \tag{6}$$

where ϵ_{ijk} is the Levi-Civita alternating tensor, and J is the moment of inertia tensor of the Earth, which may be represented as the following sum of four distinct terms [see also *Ricard et al.*, 1993]:

$$\boldsymbol{J} = \boldsymbol{J}^s + \boldsymbol{J}^\omega + \boldsymbol{J}^c + I \tag{7}$$

where $\boldsymbol{J}^s$, $\boldsymbol{J}^\omega$, $\boldsymbol{J}^c$, and I are, respectively, the moment of inertia of the assumed spherically symmetric Earth; that due to the rotation of the Earth prior to perturbation (including any process that redistributes Earth mass); that due to mantle convection, tectonics, or other geophysical processes; and that due to the perturbations associated with ice sheet growth and decay. Assuming that the rotation of the Earth is steady prior to perturbation, $\boldsymbol{J}^\omega$ can be calculated on the basis of *Munk and MacDonald* [1960a, equation (5.3.5)], namely,

$$\begin{aligned} J_{ij}^\omega &= \frac{a^5}{3G}k_2^T(t)*\left(\omega_i(t)\omega_j(t) - \frac{1}{3}\omega^2(t)\delta_{ij}\right)\Big|_{t=\infty} \\ &= \frac{a^5}{3G}k_F^T\left(\omega_i(t_{0^-})\omega_j(t_{0^-}) - \frac{1}{3}\omega^2(t_{0^-})\delta_{ij}\right) \end{aligned} \tag{8}$$

where $k_2^T(t)$ is the degree 2 tidal Love number [*Peltier*, 1982; *Wu and Peltier*, 1984], and k_F^T is the fluid limit of $k_2^T(t)$, which is defined as [e.g., *Peltier and Luthcke*, 2009; *Cambiotti et al.*, 2010]

$$k_F^T = \lim_{t\to\infty} k_2^T * H(t) \tag{9}$$

where $H(t)$ is the Heaviside step function. G is the gravitational constant, and t_{0^-} is the instant prior to application of a perturbation when $\omega_1 = \omega_2 = 0$ and $\omega_3 = \Omega$, so that

$$\begin{aligned} \boldsymbol{J} = J^s\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} + \frac{1}{3}\frac{k_F^T a^5\Omega^2}{3G}\begin{bmatrix}-1&0&0\\0&-1&0\\0&0&2\end{bmatrix} \\ + \begin{bmatrix}J_{11}^c & J_{12}^c & J_{13}^c\\ J_{21}^c & J_{22}^c & J_{23}^c\\ J_{31}^c & J_{32}^c & J_{33}^c\end{bmatrix} \end{aligned} \tag{10}$$

[15] In the traditional theory [*Peltier*, 1982; *Sabadini et al.*, 1982; *Wu and Peltier*, 1984] that is usually employed for the calculation of the impact of Earth's rotation on sea level change during ice ages, the moment of inertia $\boldsymbol{J}^c$ is usually ignored due to its presumably long time scale relative to that of the late Quaternary glacial cycles. Even in this case, however, there may exist a small though perhaps nonnegligible impact of the convection process which acts through the action of an excess ellipticity of figure supported by the stress field associated with the convection process. However, on the time scale of snowball Earth glaciations, $\boldsymbol{J}^c$ may change substantially. Nonzero nondiagonal components of $\boldsymbol{J}^c$ (equivalent to saying that the maximum principle moment of inertia is now not aligned with the rotational axis of the Earth) can of course cause significant TPW to occur [e.g., *Steinberger and O'Connell*, 1997].

[16] Since the process of mantle convection (as well as the plate tectonic phenomenon associated with it) during the Neoproterozoic is essentially unconstrained, we are unable to accurately determine how it might have been affecting Earth's rotation during this epoch of time. It could have conceivably shifted the rotational axis of the Earth either in the same direction as that due to ice sheet loading or in the opposite direction, the former of which is probably more likely based on the paleocontinental reconstructions and speculations of *Li et al.* [2004, 2008]. Here we will consider only a very simple case in which $\boldsymbol{J}^c$ is assumed to have only diagonal

components and will furthermore assume that $J^c_{11} = J^c_{22}$. As will be clearly shown below [*Cambiotti et al*., 2010], this could contribute to the excess ellipticity of the Earth if $J^c_{11} = J^c_{22} < J^c_{33}$. In this most probably overly simplified case, the total moment of inertia of the Earth before perturbation due to ice sheet growth and decay also has the diagonal form

$$\boldsymbol{J} = \begin{bmatrix} A & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & C \end{bmatrix} \tag{11a}$$

where

$$A = J^s - \frac{1}{3}\frac{k_F^T a^5 \Omega^2}{3G} + J^c_{11} \tag{11b}$$

$$B = J^s - \frac{1}{3}\frac{k_F^T a^5 \Omega^2}{3G} + J^c_{22} \tag{11c}$$

$$C = J^s + \frac{2}{3}\frac{k_F^T a^5 \Omega^2}{3G} + J^c_{33} \tag{11d}$$

where $A = B$ since we have assumed that $J^c_{11} = J^c_{22}$. If density heterogeneity due to geophysical processes such as mantle convection were absent, i.e., $\boldsymbol{J}^C = 0$, or were such that $J^c_{11} = J^c_{22} = J^c_{33}$, then the fluid limit Love number k_F^T would be related to $\boldsymbol{J}$ from equation (10), as

$$k_F^T = \frac{3G}{a^5\Omega^2}(J_{33} - J_{11}) \tag{12}$$

[17] Similarly, we define k_f^T when $J^c_{11} = J^c_{22} < J^c_{33}$ as

$$\begin{aligned} k_f^T &= \frac{3G}{a^5\Omega^2}(J_{33} - J_{11}) \\ &= \frac{3G}{a^5\Omega^2}(C - A) \\ &= k_F^T + \frac{3G}{a^5\Omega^2}\left(J^C_{33} - J^C_{11}\right) \\ &= k_F^T + \beta \end{aligned} \tag{13}$$

k_F^T or k_f^T is clearly related to the flattening of the Earth. In fact, the k_F^T predicted for a spherically symmetric Maxwell model of the tidally unperturbed Earth [*Wu and Peltier*, 1984] is very close to that observed through astronomical inferences of the moments of inertia of the present-day Earth [e.g., *Yoder*, 1995; *Chambat and Valette*, 2001], the difference being less than 1% [*Peltier and Luthcke*, 2009], with the observed value being the larger. Equation (13) demonstrates that mantle convection could have contributed (if $\beta > 0$) to this difference, although it could equally well be due to observational errors, imprecise density structure employed in the theoretical models, or that the Maxwell viscoelastic model is insufficiently general to describe the deformation of the real Earth [*Munk and MacDonald*, 1960b]. However, it is not unreasonable to assume that there will always be some, perhaps extremely small, positive mass anomaly in the equatorial plane since TPW tends to shift any positive mass anomaly toward that plane.

[18] To investigate the influence of excess ellipticity on the rotation of the Earth, *Peltier and Luthcke* [2009, equation (17b)] introduced a parameter ϵ defined as $\epsilon = \left(k^T_{F,\mathrm{obs}} - k^T_F\right)/k^T_{F,\mathrm{obs}} \approx k^T_{F,\mathrm{obs}} - k^T_F$ since $k^T_{F,\mathrm{obs}} \sim 0.94 = O(1)$. This parameter is therefore similar to the definition of β in *Mitrovica et al*. [2005] and *Cambiotti et al*. [2010]. Here we will continue to employ the symbol ϵ and its definition above to represent the possible extra flattening of Earth that the tidally forced spherically symmetric Maxwell Earth model is unable to capture.

[19] The perturbation to the moment of inertia $\boldsymbol{I}$ associated with ice sheet loading and unloading has two components: the first part is that due to the direct influence of the redistribution of mass between the oceans and continents, including that due to isostatic adjustment of the solid Earth under this change of surface load, and the second component is that due to the adjustment of the Earth to the perturbed rotational potential. Following *Peltier* [1982], *Wu and Peltier* [1984], and *Peltier* [2007]

$$\boldsymbol{I} = \boldsymbol{I}^{\mathrm{GIA}} + \boldsymbol{I}^{\mathrm{Rot}} \tag{14}$$

where

$$\boldsymbol{I}^{\mathrm{GIA}} = \left(1 + k_2^L\right) * \boldsymbol{I}^{\mathrm{Rigid}} \tag{15}$$

[20] In equation (15), the only nonzero components of $\boldsymbol{I}^{\mathrm{Rigid}}$ that need to be retained are $I^{\mathrm{Rigid}}_{13}, I^{\mathrm{Rigid}}_{23}$, and I^{Rigid}_{33}, which represent the direct contribution to the moment of inertia due to the variation of the ice sheet mass that would exist if the Earth were rigid. The second component due to the changing centrifugal potential can be obtained from equation (8) by calculating the change of J^ω_{ij} as a consequence of changing angular velocity changes with time as in equation (5), i.e.,

$$\boldsymbol{I}^{\mathrm{Rot}} = \frac{k_2^T(C - A)}{k_f^T} * \begin{bmatrix} m_1^2 & m_1 m_2 & m_1 \\ m_1 m_2 & m_2^2 & m_2 \\ m_1 & m_2 & 2m_3' \end{bmatrix} \tag{16}$$

where m_3' is the change of the rotational rate of the Earth. If the second-order terms are neglected and because $m_1, m_2 \ll 1$ and usually $m_3' \ll m_1$ or m_2 [*Han and Wahr*, 1989], equation (16) becomes

$$\boldsymbol{I}^{\mathrm{Rot}} = \frac{k_2^T(C - A)}{k_f^T} * \begin{bmatrix} 0 & 0 & m_1 \\ 0 & 0 & m_2 \\ m_1 & m_2 & 0 \end{bmatrix} \tag{17}$$

[21] Substituting equations (5), (7), (11), (15), and (17) into (6) gives the Liouville equations [*Munk and MacDonald*, 1960a] as

$$\frac{i\frac{d\overline{m}}{dt}}{\Omega\left(\frac{C-A}{A}\right)} + \overline{m} = \frac{1}{C - A}\left(\overline{I} - \frac{i\frac{d\overline{I}}{dt}}{\Omega}\right) \tag{18a}$$

$$\frac{\dot{m}_3'}{} = \frac{-\dot{I}_{33}}{C} \tag{18b}$$

where $i = \sqrt{-1}$; and $\overline{m}$ and $\overline{\boldsymbol{I}}$ are equal to $m_1 + im_2$ and $I_{13} + iI_{23}$, respectively. Equation (18) has analytical solutions for a spherically symmetrical layered Earth if the Chandler wobble is neglected [*Peltier*, 1982; *Wu and Peltier*, 1984; *Peltier and Luthcke*, 2009], as

$$m_1(t) = \frac{1}{1+\epsilon+\epsilon'}\left(\frac{\Omega}{A\sigma_0}\right)\left[\left(1+k_2^{\mathrm{LE}}\right)I_{13}^{\mathrm{Rigid}}(t) + \sum_{i=1}^{N} E_i' e^{-\kappa_i t} * I_{13}^{\mathrm{Rigid}}(t)\right] \quad (19a)$$

$$m_2(t) = \frac{1}{1+\epsilon+\epsilon'}\left(\frac{\Omega}{A\sigma_0}\right)\left[\left(1+k_2^{\mathrm{LE}}\right)I_{23}^{\mathrm{Rigid}}(t) + \sum_{i=1}^{N} E_i' e^{-\kappa_i t} * I_{23}^{\mathrm{Rigid}}(t)\right] \quad (19b)$$

$$m_3'(t) = -\frac{1}{C}\left[\left(1+k_2^{\mathrm{LE}}\right)I_{33}^{\mathrm{Rigid}}(t) + \sum_{i=1}^{N} q_i^{\ell} e^{-s_i^{\ell} t} * I_{33}^{\mathrm{Rigid}}(t)\right] \quad (19c)$$

[22] Equations (19a) and (19b) describe the change of the orientation of the spin axis of the Earth (or TPW) relative to the mean mantle, while equation (19c) describes the induced change of the axial rotation rate of the Earth. Equations (19a) and (19b) are the same as *Peltier and Luthcke* [2009, equation (27)], where σ_0 is the Chandler wobble frequency of a viscoelastic Earth, i.e.,

$$\sigma_0 = \frac{k_F^T - k_2^{\mathrm{TE}}}{k_F^T} \cdot \frac{(C-A)\Omega}{C} \quad (20)$$

ϵ and ϵ' are defined as

$$\epsilon = 1 - \frac{k_F^T}{k_f^T}, \quad \epsilon' = \epsilon \frac{k_F^T}{k_F^T - k_2^{\mathrm{TE}}} \quad (21)$$

respectively. k_2^{LE} in equation (19) and k_2^{TE} in equation (20) are elastic Love numbers for degree 2 surface mass loading and tidal loading, respectively. E_i' and κ_i in equations (19a) and (19b) are the amplitude and the time constant for the *i*th rotational mode of the Earth, respectively, the detailed form of which is provided in *Peltier and Luthcke* [2009, equations (27c) and (23a)] and are not elaborated here. In contrast, q_i^{ℓ} and s_i^{ℓ} in equation (19c) are the amplitude and the time constant for the *i*th mode of viscous gravitational relaxation [see *Peltier and Luthcke*, 2009, equation (6a)].

[23] It is worth noting that m_3' in equation (19c) is used in the traditional linear theory [*Sabadini and Peltier*, 1981; *Wu and Peltier*, 1982] to represent only the change of rotational rate (or LOD) of the Earth. When the amount of TPW is very small, m_3 defined in equation (5) is approximately equal to m_3', and they are not usually distinguished. However, the actual relationship between m_3 and m_3' is $m_3 = m_3' + \left(\sqrt{1-m_1^2-m_2^2}-1\right)$, with the second term of the right-hand side larger than m_3' once the total polar wander exceeds ~0.01°. Therefore m_3 can be much larger than m_3' even during the Pleistocene glacial cycles, but because m_3 is still negligible compared to m_1 or m_2, this imprecise representation does not make any difference in the calculation of relative sea level change. When the TPW angle is large, however, as it will be in the experiments in which TPW is prescribed, the distinction between m_3 and m_3' must be made explicitly.

[24] The procedure for calculating the influence of rotational feedback on sea level is therefore, first, to calculate the moment of inertia $\boldsymbol{I}^{\mathrm{Rigid}}$ and, then, to calculate the perturbation to the rotational state of the Earth from equation (19), which can be used to obtain the perturbation to the rotational potential from equations (3) and (4). Finally, the relative sea level change can be calculated by solving the sea level equation (1). Because the relative sea level change and the change in the rotational state are fully coupled, the procedure above needs to be iterated to obtain the correct sea level change for each time step [*Peltier*, 1998] using the extension of the pseudospectral method [*Mitrovica and Peltier*, 1991], which incorporates the influence of rotational feedback. For present purposes, the solutions thereby produced are truncated to a maximum spherical harmonic degree and order of 256.

3. Qualitative Discussion of the Expected Impact of Neoproterozoic Glaciation on Earth's Rotation and Sea Level and Design of the Numerical Experiments to be Performed

[25] In this section, we will provide initial qualitative discussion of the impacts that the following section of the paper will discuss in quantitative detail. We will also discuss the design of the numerical experiments to be performed in order to assess the sensitivity of rotation-induced impacts on relative sea level change to changes in the properties of the Earth model required for the calculation of these impacts.

3.1. The Impact on Sea Level History of a Shorter Neoproterozoic LOD

[26] Due to tidal dissipation originating primarily from the gravitational interaction between the Earth and the Moon, the Earth's rate of rotation has been decreasing during the course of its evolution (e.g., see *Lambeck*, [1980] for a review). Therefore, the length of day (LOD) was shorter during the Neoproterozoic. Sedimentary evidence strongly suggests that the LOD in the period between the Sturtian and Marinoan glaciations at ~720 and ~650 Ma was approximately 22 h [*Schmidt and Williams*, 1995]. Because more rapid rotation will act so as to increase the rotational stability of the planet, in what follows, we test four cases in which the LOD is varied through the sequence 21, 22, 23, and 24 h.

[27] Without loss of generality by assuming $\boldsymbol{J}^C = 0$, thus $\epsilon = \epsilon' = 0$, the stabilizing effect of more rapid rotation is then clearly embedded in the factor $\frac{\Omega}{A\sigma_0}$ of equations (19a) and (19b), which can be rewritten using equation (20) as

$$\frac{\Omega}{A\sigma_0} = \frac{k_F^T}{k_F^T - k_2^{\mathrm{TE}}} \cdot \frac{C}{A(C-A)} \quad (22)$$

where $\frac{k_F^T}{k_F^T - k_2^{\mathrm{TE}}}$ is constant if the viscoelastic structure of the Earth is assumed fixed, whereas the factor $\frac{C}{A(C-A)}$ is dependent on the rotation rate of the Earth and can be determined using equation (11). The variation of $\frac{C}{A(C-A)}$ with LOD is shown in Figure 1. Clearly, when the length of day decreases or the rate of axial rotation increases, the value of $\frac{C}{A(C-A)}$ decreases; therefore, $\frac{\Omega}{A\sigma_0}$ decreases. According to equations (19a) and (19b), this implies that when the rotation rate is higher, the orientation of the spin axis changes less in response to a fixed perturbation to the moment of inertia tensor. For example, when the LOD decreases from the present-day value of 24 to 22 h, the value of $\frac{\Omega}{A\sigma_0}$ decreases by ~16%. In response, the sea level change

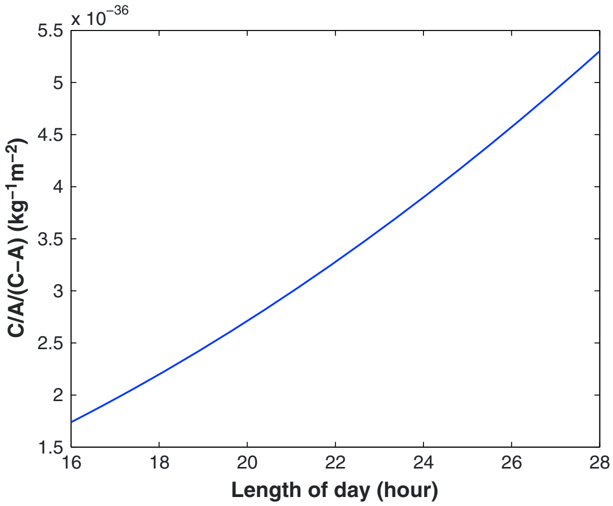

Figure 1. Relationship between $\frac{C}{A(C-A)}$ and length of day.

associated with a fixed perturbation is expected to be smaller when the rotation rate of the Earth is higher.

3.2. Influence of Excess Ellipticity

[28] Nonzero ϵ also acts to stabilize the rotation of Earth. This is easily seen from the factor $\frac{1}{1+\epsilon+\epsilon'}$ in equations (19a) and (19b), which becomes smaller as ϵ becomes larger. It has been suggested by some authors [*Mitrovica et al.*, 2005; *Cambiotti et al.*, 2011; *Mitrovica and Wahr*, 2011] that the value of β (equation (13)) for the present-day Earth may be approximately 0.008 or even larger. In *Peltier and Luthcke* [2009], this corresponds closest to the case $\epsilon = 0.00769$ and $k^T_{F,\text{obs}} = 0.9414$, a case in which the appropriate value for the elastic thickness of the lithosphere is taken to be zero in the limit of long time in computing the response of the planet to tidal forcing. Although the results in *Peltier and Luthcke* [2009, Figure 6] clearly establish that a value of ϵ of this magnitude has no significant influence on the fit of the model of late Quaternary glaciation and deglaciation to the constraints on polar wander speed and direction in the period prior to the modern global warming era [*Roy and Peltier*, 2011], it may nevertheless have a significant impact on the very long time scales associated with Neoproterozoic glaciation events. Therefore, in what follows, we will explore values of ϵ ranging from 0 to 0.01 in steps of 0.002. When ϵ is fixed to, for example, a value of 0.008, the factor $\frac{1}{1+\epsilon+\epsilon'}$ will be approximately 1.5% less than unity; therefore, it might be expected that the maximum influence of ϵ on polar wander would be negligibly small. However, the influence of ϵ also appears in the calculation of the factors E_i' and κ_i [see *Peltier and Luthcke*, 2009, equation (22)], so that the actual influence of ϵ on polar wander could be significant on long time scales. Detailed quantitative analysis is clearly necessary, which is our goal in the quantitative results section to follow.

[29] The influence of ϵ is readily understood on physical grounds. A nonzero ϵ acts as an excess permanent mass load on the equator. Therefore, any other external surface mass loading event (e.g., due to continental ice sheet growth and decay) must compete with this load in order to perturb the rotation axis of the Earth. In the case of $\epsilon = 0$, any constant external loading, however small, will perturb the rotational axis of the Earth to a large degree (after a sufficiently long time) until it is located at the new (rotational) equator. A nonzero value of ϵ will render this unlikely to occur unless the applied load redistribution is much larger than that equivalent to ϵ. Once the external load is removed, the rotational axis will simply return to its original position. This has been described schematically in *Mitrovica et al.* [2005].

[30] We can actually estimate the equivalent mass load for a nonzero ϵ by using equation (13) and substituting β with ϵ. For a value of 0.008 for ϵ, the value of $J^c_{33} - J^c_{11}$ is 2.64×10^{33} kg m^2 when the LOD is assumed to be 22 h. Assuming that the hypothesized mass is evenly distributed along the equator, then this mass can be calculated according to the definition of J^c_{33} and J^c_{11}, as $M = 2\left(J^c_{33} - J^c_{11}\right)/a^2 = 1.3 \times 10^{20}$ kg. Converting this mass to volume of ice, we get $143 \times 10^6\,km^3$, which is a very large number, comparable to the total land ice volume during a snowball Earth glaciation (~$400 \times 10^6\,km^3$, see Figure 3). More importantly, this hypothesized mass load will not be compensated by the deformation of the solid Earth, unlike the load associated with real ice sheets, more than 90% of which will be compensated after a few tens (or hundreds, depending on the viscosity of mantle) of thousands of years. Even for $\epsilon = 0.002$, the hypothesized mass load will be 0.33×10^{20} kg or $36 \times 10^6\,km^3$ in terms of ice volume, which is similar to the size of the Antarctic ice sheet but now placed (evenly) at the equator. This mass load is still comparable to that of snowball Earth ice sheets after being compensated for a few tens of thousands of years and much more in excess of it after 1 Myr. Therefore, the ice load may perturb the rotational axis rapidly initially before it is compensated due to the isostatic adjustment process; as it is being compensated with time, the hypothesized mass load associated with the excess ellipticity will become more and more significant and will act so as to drive the rotation axis back toward its original position. Of course, the final position of the rotational axis should still slightly deviate from its original position and will be determined by the compensated ice load and the hypothesized mass load associated with the excess ellipticity.

3.3. Continental Configurations and Model Ice Sheets to Be Employed as Basis for Quantitative Analysis

[31] As basis for the quantitative analyses of the sea level changes expected due to Neoproterozoic glaciations, we will employ the same realistic continental configurations as in *Liu and Peltier* [2010, 2011, 2013] for the purpose of our analyses. Specifically, we will employ the continental reconstructions for 720 Ma (Figure 2a) of *Li et al.* [2008] and for 570 Ma (Figure 2b) of *Dalziel* [1997] for the Sturtian and Marinoan glaciations, respectively. Also similar to *Liu and Peltier* [2013], the continents will be assumed to be free of topography, with a freeboard of only 10 m (i.e., the surface of the land is taken to be everywhere10 m above the initial ocean surface, which is the reference (geoidal) surface). The ocean floor in these models is assumed to lie at a uniform depth of 4000 m below the reference surface.

[32] Unlike *Liu and Peltier* [2013], however, the loading history of the ice sheets may have a nonnegligible influence on the final "equilibrium" state of the rotation of the Earth

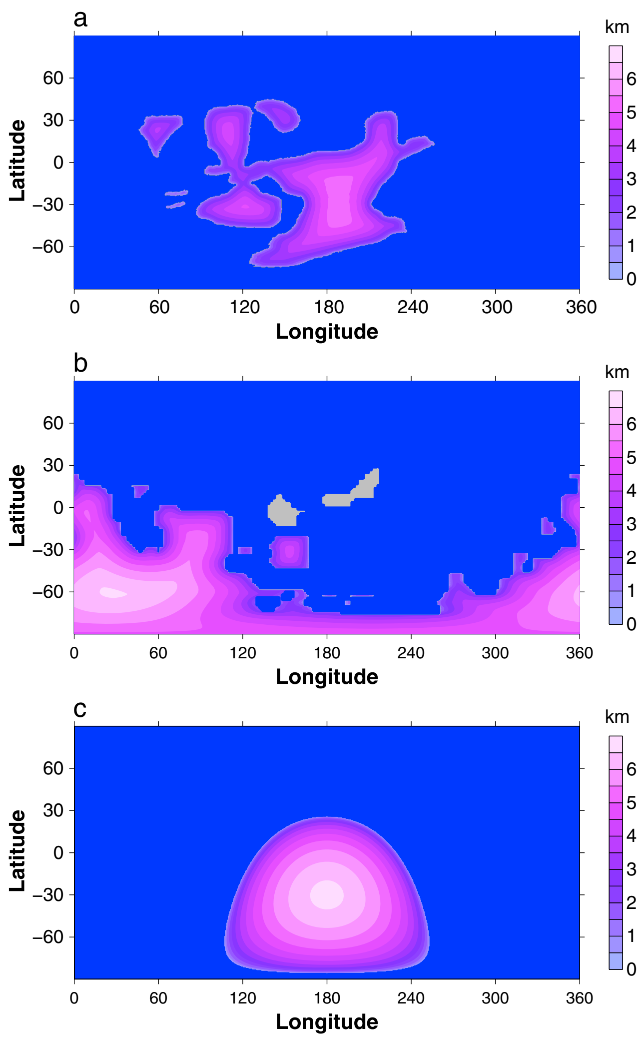

Figure 2. Continental configuration (grey) and ice thickness (blue pink contour) distributions for (a) 720 Ma, (b) 570 Ma, and (c) a circular ideal supercontinent centered at 30°S. The grey area in Figure 2b is the continental pieces that are not covered by ice.

and, thus, on sea level change. In most of the calculations to be described in what follows, we will not continue our integrations into this equilibrium state as the adjustment times prove to be extremely long in many cases, as will be discussed further below. We will therefore need to explicitly examine the influence of ice sheet loading history. Figure 3 shows examples of time series of ice volume change during the formation of the snowball Earth state for both 720 and 570 Ma continental configurations. The ice sheet history is obtained by employing an ice sheet coupled energy balance climate model (EBM) (the UofT Glacial Systems Model (GSM)) [*Deblonde and Peltier*, 1990; *Deblonde et al.*, 1992; *Tarasov and Peltier*, 1997, 1999] by decreasing the atmospheric CO_2 concentration very slowly. The ice volume changes are those predicted to occur as the system executes the transition into the soft snowball state during its traverse of the hysteresis loop of steady state solutions when the carbon dioxide level of the atmosphere is reduced below the critical value for which this transition occurs [e.g., see *Liu and Peltier*, 2013, Figure 1]. For each of the continental configurations, we will consider three ice loading histories, the major difference among them being the duration of the snowball Earth formation event. In one of the ice loading histories, the ice sheets of a soft snowball state are assumed to be emplaced on the continents instantaneously, while in the other two, the duration of the transition is assumed to be either 500 kyr or 1 Myr. The duration of the soft snowball formation event in these models was controlled by tuning the rate at which the atmospheric CO_2 concentration decreases in separate simulations with the ice sheet coupled energy balance climate model (see above). There is no need to separately test the change in the ice sheet loading event in the case of formation of a hard snowball because the resulting continental ice mass does not differ significantly from that of the soft snowball. The ice thickness distributions at the end of the ice loading histories for the 720 and 570 Ma continental configurations are shown in Figures 2a and 2b, respectively.

[33] Because the latitudinal position of the continents and their ice sheet loads will affect the values of the perturbations to the moment of inertia tensor I^{Rigid} (equations (15) and (19)), particularly if the load is either at the equator or on one of the poles where the value of I_{13}^{Rigid} and I_{23}^{Rigid} would be zero, the polar wander components m_1 and m_2 should also be zero according to equation (19). Moreover, I_{13}^{Rigid} and I_{23}^{Rigid} should be largest when the load is located at 45° latitude. This enables a useful test case of the validity of the software we have developed to solve this problem and is best performed by employing a geometrically simple model of continental glaciation. For this particular purpose, we will therefore employ an ideal circular supercontinent located at 180°E, but whose centroid is a function of latitude. Similar to the analyses discussed in *Liu and Peltier* [2013], the radius of the simple model continent will be fixed to 56°. The ice sheet that would

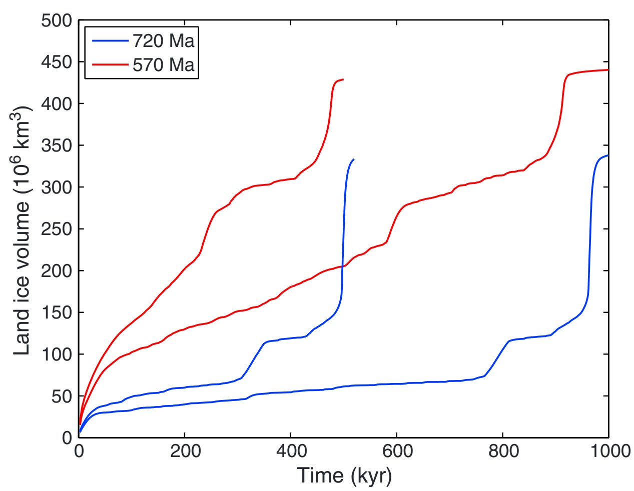

Figure 3. Ice loading histories employed for 720 Ma (blue curves) and 570 Ma (red curves) continental configurations, respectively. These loading histories are obtained by a coupled EBM and ice sheet model, in which the CO_2 concentration is decreased at different rates so that a soft snowball Earth is formed at 500 kyr (short curves) and 1 Myr (long curves), respectively. For the 720 Ma continental configuration, the 500 kyr history is actually 520 kyr (short blue curve).

exist in a snowball Earth state is also obtained using the same ice sheet model (UofT GSM) coupled to an energy balance model as above. An example of ice thickness and extent for the supercontinent centered on 30° south latitude is shown in Figure 2c. This ice sheet is emplaced on the continent gradually over a period of 500 kyr. To be certain that the mass center of the ice sheet coincides with that of the continent at all times, it is assumed that ice grows uniformly over the continent, and the thickness increases linearly (at different rates locally) with time until it reaches the maximum at 500 kyr.

[34] In *Liu and Peltier* [2013], in which the influence of rotational feedback was not considered, the sea level was shown to achieve a close to equilibrium level within a few hundred thousand years after completion of the continental glaciation event. When the influence of Earth's rotation is included, however, the rotational state of the Earth may continue to change on a time scale in excess of 10 Myr after the ice loading event is completed. This is because the continental loading event will not become isostatically adjusted even on a time scale of this order so long as the continental plate on which the ice sheet has been emplaced continues to be characterized by a finite elastic lithospheric thickness. It is therefore not possible for the system to achieve a true equilibrium state until the loaded continent lies on the equator (in the case $\epsilon = 0$). In cases in which ϵ is not equal to zero, the time scale is so long that the time dependence of other processes such as mantle convection would render the calculation of limited physical meaning.

[35] Moreover, the rotational axis of the Earth could be displaced by a large angle over only a few million years, which would violate the assumption that m_1, m_2, and m_3 must remain $<<1$ (e.g., in (17)). If we take the maximum allowable value of m_1 or m_2 to be 0.1, then the maximum displacement (calculated by $\sin^{-1}\sqrt{m_1^2 + m_2^2}$) of the rotational axis away from its original position is 8.1°. However, an accurate result is not guaranteed even when m_1, m_2, and m_3 are very small. For example, consider a load whose centroid is offset from the equator by only 2°; this will cause the rotation axis of the Earth to move away from the load. Physically, the movement of the rotational axis will cease after being displaced by 2° because the load will then be at the position of the new rotational equator. However, the formulation above always calculates $\boldsymbol{I}^{\mathbf{Rigid}}$ (implicitly) as if the rotational axis is still at the original position. However, unreasonable as it may seem, this has a negligible effect on the sea level calculations to be performed here because polar wander speed is very small ($<$0.01°/Myr) when the load is close to the equator. As will be seen in the analyses to follow, the TPW for the realistic continental configurations and associated ice sheets will be fast ($>$1°/Myr but usually $<$8°/Myr) during the formation of a snowball Earth (the duration is assumed to be $\leq$1 Myr) and then becomes steady rather quickly with a relatively small TPW speed (around 0.5°/Myr and 1°/Myr for 720 and 570 Ma continental configurations, respectively, but for which the rates will also be dependent on the viscosity of the mantle). Therefore, the total amount of TPW rarely becomes larger than 8.1° during the first few million years, and if the lower mantle viscosity is high or there is nonzero excess ellipticity, it never reaches 8.1° even in a period as long as 10 Myr.

[36] Because the TPW speed will be steady, even though it decreases greatly soon after snowball formation (a few hundred thousand years) due to continuous compensation by the mantle of the ice loading event, the glaciated supercontinent will continue to drift to lower latitude. This drift could be accompanied by a continuous diminution of continental ice mass since, according to the model of *Peltier et al.* [2007], the system must eventually deglaciate spontaneously. The observational constraints fix the glacial period to less than a few tens of millions of years at most. However, an upper bound estimate on the extent of relative sea level change due to the changing rotational state on very long time scale may be provided by employing a prescribed constant TPW speed. Models of this kind are described in what follows.

3.4. Sea Level Changes Resulting From a Prescribed Change in the Rotational State of the Earth

[37] To test how relative sea level might be expected to change as a function of the depth dependence of viscosity and of the magnitude of the perturbations to the rotational state, it proves useful to simply prescribe a particular perturbation that is not connected to any particular glaciation history.

[38] Our purpose here is not only to estimate how relative sea level changes after TPW enters a steady state as described above but also to develop a qualitative understanding of how the relative sea level is expected to change in response to the action of a constant or a variable rate of TPW over time scales on the order of or longer than the duration of a snowball Earth glaciation event. Such background TPW may also be thought to be associated with mantle convection and may well be coeval with the occurrence of a snowball Earth event.

[39] We will test the prescribed changes in both the TPW [m_1 and m_2 in equation (19)] and the axial rotation rate (m_3' in equation (19)). For the case of TPW, two sets of experiments will be performed. In the first set, the TPW speed will be assumed to be time independent (i.e., m_1 and m_2 increase with time at a constant rate), and the TPW direction will be assumed to be aligned with 0°E (i.e., only m_1 changes with time, and m_2 is identically 0), consistent with ice sheet loading in the Southern Hemisphere that is centered at 180°E. Results will be obtained for TPW speed in the sequence 1°/Myr, 2°/Myr, 4°/Myr, 6°/Myr, and 8°/Myr. In the second set of experiments, the TPW speed will be assumed to have a higher constant value $V_{1,\mathrm{tpw}}$ of either 4°/Myr or 8°/Myr (comparable to the maximum TPW speed induced by the snowball Earth ice sheets for the 720 and 570 Ma continental configurations, respectively) during the first million years and a lower constant value $V_{2,\mathrm{tpw}}$ of either 0.5°/Myr or 1°/Myr during the ensuing time period. In detail, the TPW speed is increased linearly from 0 to $V_{1,\mathrm{tpw}}$ during the first 300 kyr, fixed to $V_{1,\mathrm{tpw}}$ from 300 kyr to 1 Myr, then linearly decreased to $V_{2,\mathrm{tpw}}$ from 1 to 1.3 Myr, and fixed at $V_{2,\mathrm{tpw}}$ during the remainder of the simulation. This two-step perturbation to the drift rate of the rotational pole position, as will become clear later, mimics the realistic TPW time history that is actually expected to characterize the period during and after a snowball Earth formation event.

[40] In order to test the influence of a perturbation to the axial rotational rate (m_3' in equation (19)), we will apply two types of perturbations to the rotational speed, the first being a case in which its value is reduced initially at the beginning and then held fixed during the remainder of the simulation, and the second being a case in which its value

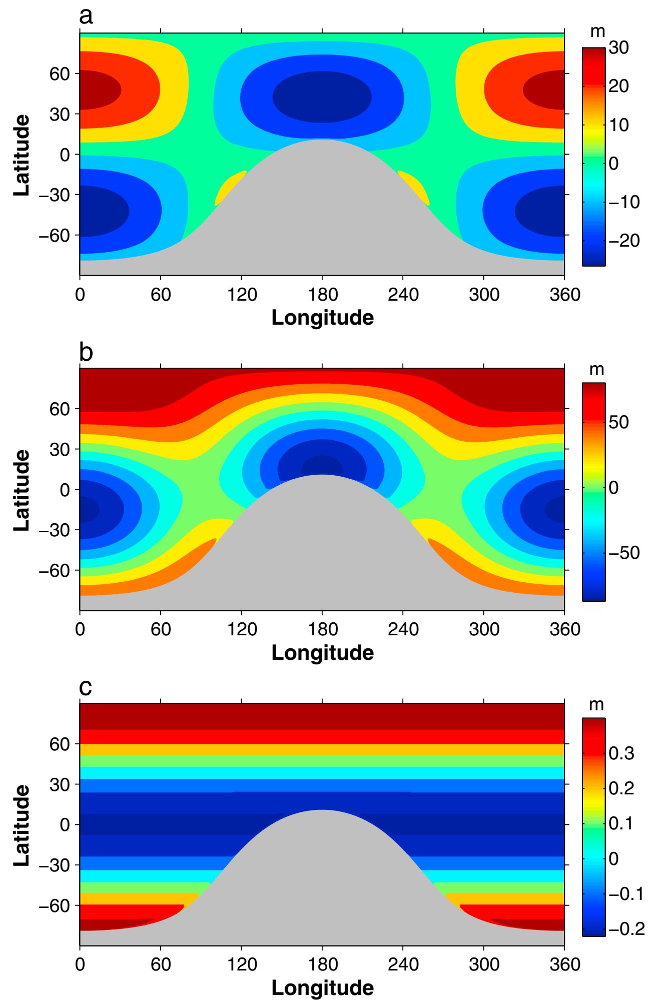

Figure 4. (a, b) Relative sea level change after 2 and 24 Myr due to TPW of constant speed of 2°/Myr. (c) Relative sea level change 500 kyr after a step reduction of the Earth's rotational rate by 0.1%. The viscosity model is VM5a. The grey area is where a circular supercontinent is located.

is continuously reduced with time to investigate the influence of long-term deceleration due to tidal friction between the Earth, the Sun, and the Moon. For the first example, a sequence of reduction of the rotational rate by 0.001%, 0.01%, 0.1%, and 1% is tested. For the second type, the rotational rate will be assumed to decrease with time at a constant rate of 1, 2, or 3 times 0.015%/Myr, where the value 0.015%/Myr is the approximate deceleration of the Earth if it is assumed to have been slowing down linearly with time during the past 600 Myr. These reductions in rotation rate will be applied as a perturbation to a base speed that is equivalent to an LOD of 22 h. In this and all previous sections, we assume that the Earth has already attained a shape that is in equilibrium with the specified LOD before ice sheets form.

[41] In all these sensitivity analyses, it is assumed that there is an ideal circular (radius of 56°) supercontinent centered at 45°S. The surface of the continent is assumed to be free of topography, with a constant elevation of 1000 m in order to ensure that no part of it will become submerged during the simulation. Further tests with realistic continental configurations do not show any significant difference in the magnitude of relative sea level change. In the section to follow, we will describe and discuss results from these experiments first.

4. Results and Discussion

4.1. Relative Sea Level Changes in Response to a Prescribed Perturbation to the Rotational State

[42] Examples of relative sea level changes due to TPW and deceleration of Earth's rotation rates are illustrated in Figures 4a and 4c, respectively. (These effects are calculated from the sea level equation in the absence of any influence of ice sheet loading and unloading.) They have the classic quadrupolar and belt structure, respectively, originally explained in *Dahlen* [1976] in the context of a different problem and as applied in *Mound and Mitrovica* [1998] to the general problem of interest to us here, although not in a Neoproterozoic context. The quadrupolar and belt structures arise due to the dominant degree 2 and order 1 and degree 2 and order 0 components of the perturbation to the centrifugal potential associated with TPW and deceleration of Earth's rate of axial rotation, respectively. Note that in

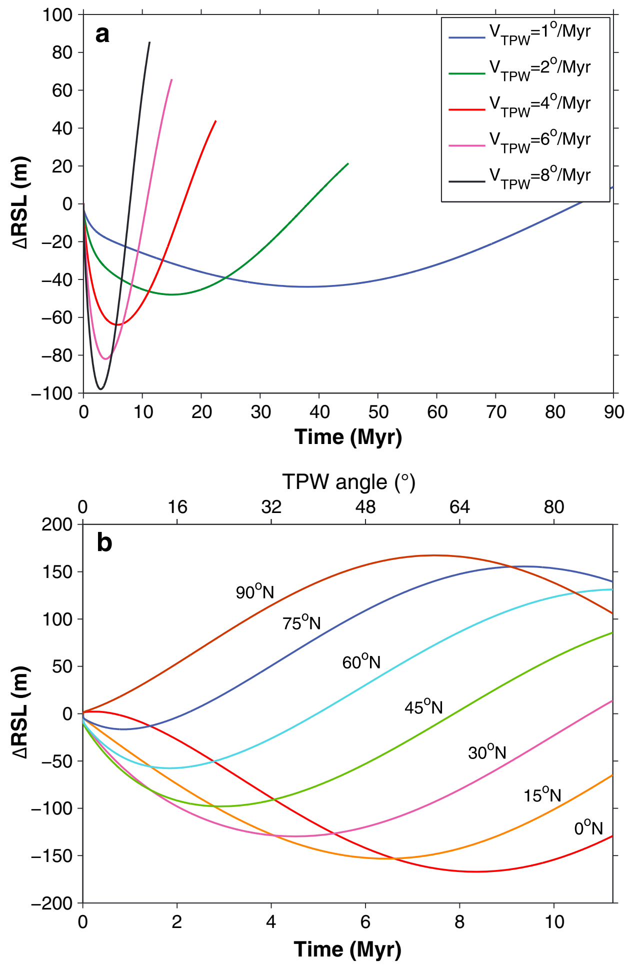

Figure 5. (a) Time series of relative sea level change at (180°E, 45°N) for different TPW speeds. (b) Time series of relative sea level change at different locations along 180°E when the TPW speed is 8°/Myr. The total TPW amount is 90°. The viscosity model for both is VM5a.

Figure 4 and all other figures herein, the longitude and the latitude are as defined before the rotational axis is perturbed or shifted, i.e., the latitude-longitude coordinates are fixed to the solid Earth from the outset. For TPW, the largest perturbation to the centrifugal potential is always located at 45°S and 45°N (crossing the great circle of the TPW movement) relative to the "instantaneous" rotational axis [e.g., *Sabadini et al.*, 1990]. After a large angular amount of TPW, the new instantaneous midlatitude will correspond to a different latitude of the original coordinate system; therefore, the quadrupolar structure will gradually disappear from the old coordinate system. Figure 4b shows the relative sea level change after the North Pole has shifted by 48° at a speed of 2°/Myr. The maximum lows now appear near the equator since according to the new rotational axis, the latitude of the original equator (at the crossing point with 180°E) is 48°N.

[43] Apparently, the cause of relative sea level change due to perturbations to the rotational state of the Earth is due to the fact that the response time of the solid Earth to a perturbed centrifugal potential is much longer than that of the redistribution of seawater (which is assumed to be instantaneous). We infer on the basis of these results that the faster the TPW, the larger the relative sea level change will be. This is indeed observed (see Figure 5a) in our calculations where a 90° shift of the pole is prescribed but with a different speed. The peak relative sea level change at 45°N is more than doubled when the TPW speed is increased from 1°/Myr to 8°/Myr. For TPW ≤ 2°/Myr, the peak relative sea level change at 45°N is only around 50 m for viscosity model VM5a. For all TPW speeds, the relative sea level change at 45°N reaches a maximum before this location is shifted to the North Pole (at the midpoint in time of the total excursion), although the centrifugal potential decreases monotonically from the equator to the pole. This is due to the fact that the values of spherical harmonic functions $Y_{21}(\lambda,\theta)$ or $Y_{2,-1}(\lambda,\theta)$ around $\theta = 90°$N or 90°S are small (meaning the loading due to centrifugal potential forcing is small), so that the relative sea level change starts to relax before reaching 90°N or 90°S.

[44] At the end of a 90° period of TPW, the midlatitude points on the great circle of the TPW movement return to the midlatitudes once more but are now on the opposite side of the great circle. If the solid Earth relaxes sufficiently quickly or TPW is sufficiently slow, the relative sea level change at these points should be approaching 0. This is only approximately reached by the slow case in which the TPW speed is 1°/Myr (blue curve in Figure 5a; also seen in Figure 6a for other viscosity models). For all other cases, the relative sea level change differs significantly from 0 (Figures 5a and 6b).

[45] It is not surprising that the maximum relative sea level change during the course of a 90° TPW event is not experienced by the midlatitude points as may be suggested at the beginning of TPW; rather, the maximum change is obtained at the poles or the equator (at the crossing points with 0°E or 180°E) because these points experience a monotonic change of centrifugal potential (see Figure 5b). For the case of a TPW speed of 8°/Myr, for viscosity model VM5a, the maximum relative sea level change experienced by the polar or equatorial points is 167 m (can be either positive or negative), much larger than the change of 98 m (also can be either positive or negative) experienced by the midlatitude points (Figure 5b). However, it must be pointed out that during the first ~32° of TPW, the point at 30°N (or 30°S) experiences the largest relative sea level change (Figure 5b), and this period is the most relevant for snowball Earth sea level as will be further discussed in what follows.

[46] Since the relative sea level change is due to the slow response of the solid Earth relative to the instantaneous redistribution of ocean water required to maintain the ocean surface as a gravitational equipotential, increasing the viscosity of any part of the Earth would be expected to increase the magnitude of relative sea level change. Figure 6 shows the time series of relative sea level change at (180°E, 45°N) for eight different viscosity models and for the slowest (1°/Myr) and fastest (8°/Myr) TPW speeds we have chosen to explicitly test. The most significant difference is obtained by changing the thickness of the elastic lithosphere. When the thickness is increased from 60 km (VM5a, solid blue line) to 120 km (VM5aL120, dashed black line), the relative sea level change is more than doubled for the TPW speed of 1°/Myr (Figure 6a). However, this sensitivity decreases as the TPW speed is increased (e.g., to 8°/Myr, Figure 6b). Reducing upper mantle viscosity by a factor of 2 (VM5b relative to VM5a) does not have a significant effect (compare the solid

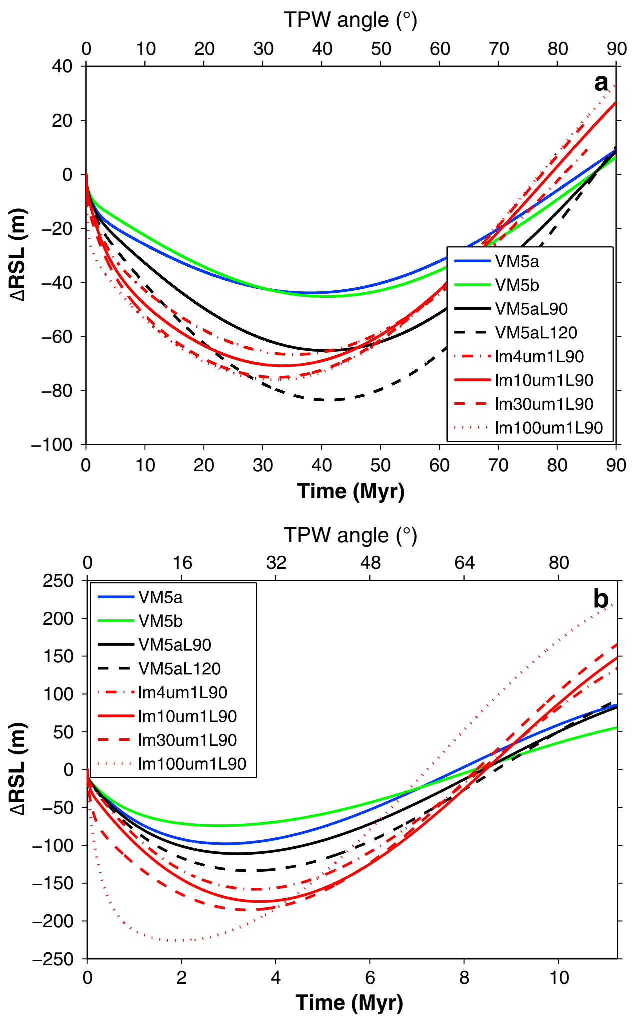

Figure 6. Time series of relative sea level change at (180° E, 45°N) for all eight different viscosity models. The TPW speeds are (a) 1°/Myr and (b) 8°/Myr.

green line with the solid blue line in Figure 6). Changing the lower mantle viscosity does not have a significant effect either (compare the red lines in Figure 6a) when the TPW speed is small, although the effect seems very large during the first 1 Myr, as first demonstrated by *Sabadini et al.* [1990]. These characteristics are the same as those obtained by *Mound et al.* [1999]. However, it is clear that when the TPW speed is large (e.g., 8°/Myr, Figure 6b), the effect of lower mantle viscosity becomes much more significant. This was apparently overlooked in *Mound et al.* [1999].

[47] A perturbation to the rate of axial rotation has only a very small effect on relative sea level. Figure 7 shows the relative sea level change at a polar point (180°E, 89°N), where the relative sea level changes most (Figure 4b) due to a step change of rotational speed of the Earth. The immediate relative sea level change after the perturbation is applied is linearly proportional to the magnitude of the perturbation. For changes of rotational speed of -0.1% (red line in Figure 7a), which is of the same order of magnitude as the total deceleration of the Earth's rotation assumed to occur in 10 Myr if it is assumed that the deceleration during the past 600 Myr is uniform, the immediate relative sea level change is ~16 m only. These relative sea level changes relax very quickly (with a time scale of 10 kyr) for viscosity model VM5a (Figure 7a). Different viscosity models do not affect the magnitude of the immediate relative sea level change, as expected, but do change the time scale of relaxation with the lower mantle viscosity having the most significant influence (Figure 7b). For a -0.01% step reduction of rotational speed, the maximum relative sea level change is smaller than 2 m for any of the viscosity models tested.

[48] When constant deceleration of the rotational speed is considered, it is found that the relative sea level change increases with time (Figure 8). However, the relative sea level change is smaller than 3 m after 10 Myr for all viscosity models (Figure 8), even for deceleration rates 3 times the rate at which Earth has been decelerating on average during the past 600 Myr if the deceleration is assumed to have been uniform. Increasing the viscosity at any depth of the mantle or increasing the thickness of the lithosphere will increase the resulting relative sea level change, similar to the role they have played in relative sea level change due to TPW, which is not surprising. Overall, Figures 7 and 8 demonstrate that the influence of deceleration of Earth's rotation on relative sea level is very small and will therefore not be considered further.

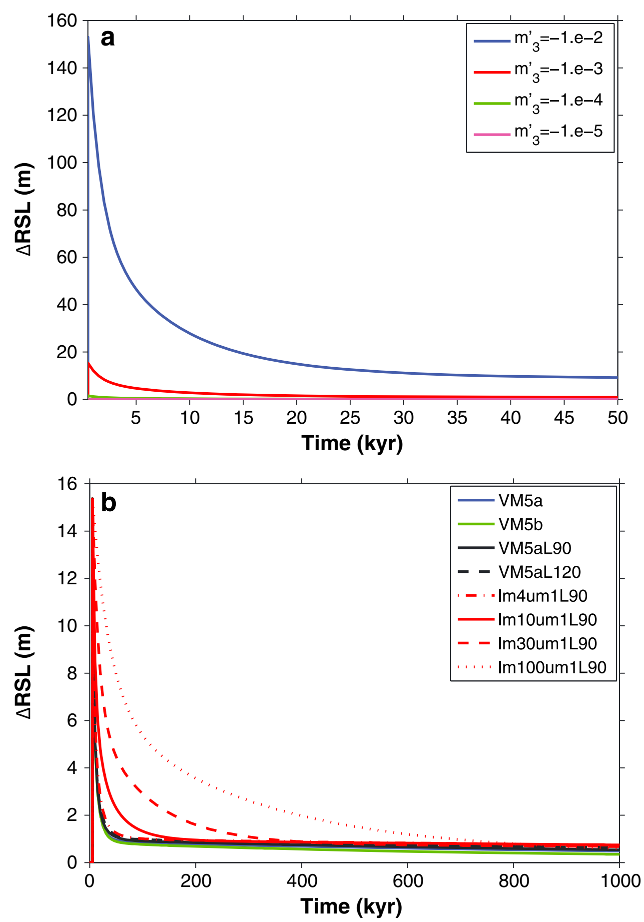

Figure 7. Time series of relative sea level change at (180° E, 89°N) due to a step change of rotational speed. (a) The rotational speed is changed by different magnitudes (as indicated by m_3'); the viscosity model is VM5a. (b) $m_3' = -1.e-3$ (i.e., the rotational speed is reduced by 0.1%), but the viscosity model is varied.

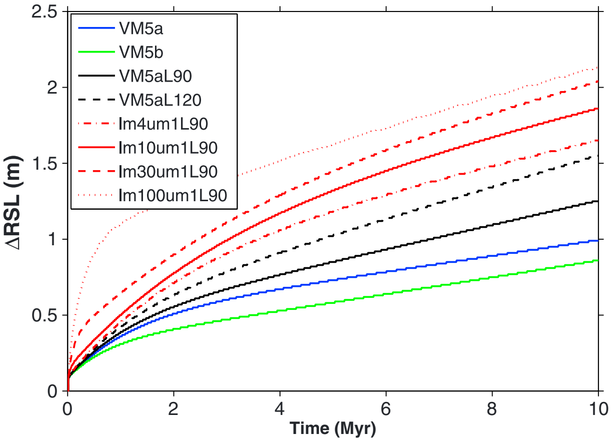

Figure 8. Time series of relative sea level change at (180° E, 89°N) due to a linear reduction of rotational speed with time. The reduction rate is 0.045%/Myr, i.e., 3 times the rate at which Earth has been decelerating during the past 600 Myr if the deceleration is assumed to be uniform.

[49] Next, we focus on perturbations to the rotational state (TPW only) of the Earth that are more relevant to snowball Earth formation, which will be shown to be characterized by a two-step change of TPW speed such that it is high during the first million years and low thereafter. The two TPW curves in Figure 9a have high speeds of 4°/Myr (blue curve) and 8°/Myr (red curve) initially and low speeds of 0.5°/Myr (blue curve) and 1°/Myr thereafter, respectively. This, as will become clear later, qualitatively resembles the characteristic TPW signature expected during snowball Earth formation and evolution for the 720 and 570 Ma continental configurations, respectively, if we assume that the snowball Earth forms in a period of 1 Myr (Figure 3) as in the explicit carbon cycle coupled climate and ice sheet models discussed in *Peltier et al*. [2007] and *Liu and Peltier* [2011]. Figures 9b and 9c show the time series of relative sea level change at (180°E, 30°N) due to these two-step changes of TPW speed. The high TPW speed causes the relative sea level to change quickly during the first million years and reaches a peak at the end of the first million years, but this peak relaxes toward 0 quickly (with time scale depending on the viscosity model) once the TPW speed is reduced, even though the speed is nonzero and in the same direction as before. Then, after a few million years, the magnitude of relative sea level change begins to increase again, almost linearly with time since 10 Myr and reaches a similar value to the previous peak value at around 20 Myr. Therefore, if the duration of the snowball Earth event is shorter than 20 Myr (which is very likely if it is a soft snowball Earth because it is much easier to melt than the hard snowball Earth) [e.g., *Liu and Peltier*, 2011], the relative sea level change due to rotational feedback is maximum around the end of the snowball Earth formation event (i.e., ~1 Myr).

[50] Location (180°E, 30°N) experiences approximately the largest peak relative sea level change as can be inferred from Figure 5b (see values in the range 1–2 Myr), and the value is highly viscosity dependent. For model VM5a, the peak value is around 30 and 65 m for a relatively slow (4°/Myr) and fast (8°/Myr) TPW speed during the first million years, respectively, but can increase to as much as 95 and 200 m (Figure 9) for viscosity model lm100um1L90, which has very high lower mantle viscosity. However, as will become clear in what follows, for the same ice loading, the induced TPW speed will be much smaller for higher lower mantle viscosity. Therefore,

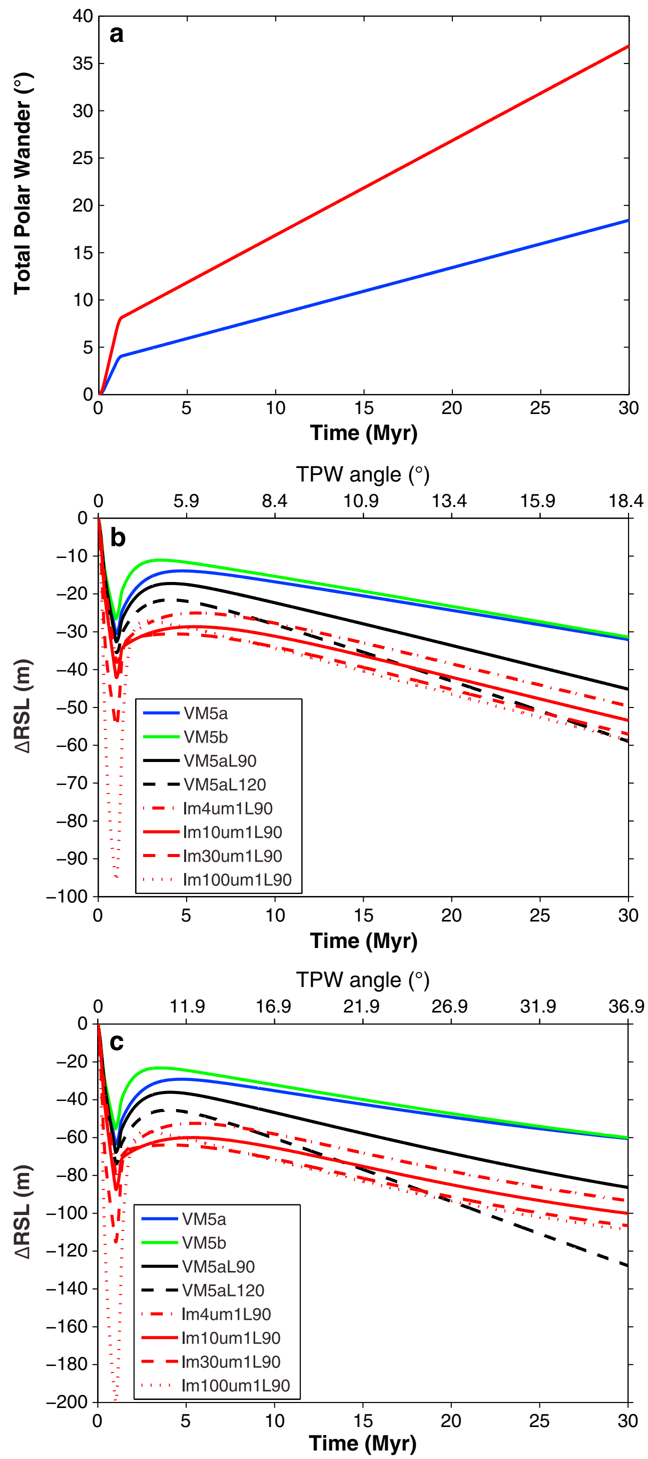

Figure 9. (a) Total amount of TPW for a two-step change of TPW speed, which is 4°/Myr (blue curve) or 8°/Myr (red curve) during the first million years and 0.5°/Myr (blue curve) or 1°/Myr (red curve) afterward. (b, c) The time series of relative sea level change at (180°E, 30°N) due to TPW prescribed by the blue and red curves in Figure 9a, respectively, for all eight different viscosity models.

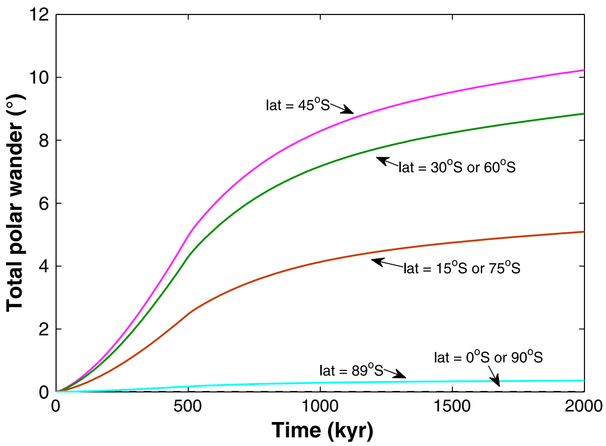

Figure 10. TPW obtained when the ideal circular supercontinent is centered at different latitudes. The viscosity model is VM5a.

the relative sea level change values for high lower mantle viscosity models in Figure 9 do not provide reasonable upper bound estimates for the snowball Earth–related sea level changes. As will also become clear in what follows, an improved upper bound estimate may be provided by the cases in which the elastic lithosphere is thick. Thicker lithosphere sustains higher TPW speed under the same ice loading conditions and, at the same time, also induces a larger relative sea level change under the same prescribed TPW speed (Figures 6 and 9). For lithosphere as thick as 120 km, the relative sea level change increases very rapidly with time even after the snowball Earth is fully formed and reaches values of ~60 m (Figure 9b) and ~127 m (Figure 9c) for the slow and fast TPW (blue and red curves in Figure 9a), respectively, at 30 Myr.

[51] As described above, the relative sea level at the optimal location, e.g., (180°E, 30°N), will increase linearly with time after 10 Myr since the onset of snowball Earth formation. The rate at which the relative sea level changes is ~1.2 or ~1.5 m/Myr if the TPW speed is maintained at 0.5°/Myr or 1°/Myr, respectively, for almost all viscosity models, except VM5aL120, which has thick lithosphere (Figure 9). Therefore, even if a snowball Earth did persist for an extended period, e.g., longer than 10 Myr, the maximum relative sea level change due to rotational feedback increases very moderately with time and can be reasonably estimated (at least the upper bound) without precise modeling of the actual rotational history (which is unknowable for the reasons previously discussed).

4.2. The Influence of the Latitudinal Position of the Supercontinent and Superimposed Ice Sheet on TPW

[52] Given the basic understanding of expected TPW impacts based on the previous analyses in this section, we will proceed to consider in detail the TPW that would be induced by a realistic snowball Earth formation and subsequent evolution event. To begin, we will investigate the impact of the initial latitudinal position of the supercontinent on the rotational response. We start by fixing the LOD to 24 h and ϵ to 0 so that the possible influence of excess ellipticity will be ignored and will assume viscosity model VM5a. As expected, if the glaciated supercontinent is initially located either at the pole or on the equator (Figure 10), TPW is negligible (should be precisely 0, but the numerical error due to the representation of the geometry of the ice sheet load makes it difficult to achieve this). Also as expected, the maximum TPW is obtained when the ice sheet is centered at 45°S (45°N). Moreover, the same displacement is obtained when the ice sheet loaded continent is centered at latitudes symmetric with respect to 45°S (e.g., 30°S and 60°S and 15°S and 75°S). These results provide a qualitative test of the validity of the code as they are consistent with the dependence of the magnitude of I_{13}^{Rigid} and I_{23}^{Rigid} on the latitude of ice sheet loading.

[53] The maximum value of m_3' due to ice loading in this calculation turns out to be on the order of 10^{-4} for all of the simulations, which is approximately 2 orders of magnitude smaller than m_1 or m_2, and is further reduced with time. Therefore, neglect of the m_3' term in equation (16) is justified. This maximum value of m_3' is also much smaller than the deceleration of Earth's rotation due to tidal friction, and even this latter effect, as has already been demonstrated in the previous section, has a negligible influence on relative sea level change. Therefore, the influence of a perturbation to Earth's rate of axial rotation will not be discussed further in what follows.

4.3. The Influence of Ice Sheet Loading History on TPW and Relative Sea Level

[54] Figure 11a illustrates the influence of ice sheet loading history on TPW for both the 720 and 570 Ma continental configurations. For the purpose of all of the simulation results shown in Figure 11a, the length of day has been fixed to 22 h, and the viscosity model is VM5a. Apparently, the more rapid the ice sheet loading event, the larger the total polar wander. The difference in total polar wander can be as large as a few degrees during the first 1.5 Myr, but the TPW due to longer ice loading history catches up quickly once the loading is completed, making the difference in total polar wander less than 0.3° at 2 Myr, and this difference remains small thereafter.

[55] It is also found that although the maximum difference in relative sea level at 1 Myr between calculations in which ice loading history lasts 1 Myr and 0 kyr (the former minus the latter) can be as large as 60 m (not shown), it is reduced to approximately 10 m at 1.5 Myr and is further reduced to less than 4 m at 2 Myr. Therefore, we may conclude that the ice sheet loading history is unimportant if our interest is in the sea level change produced more than a few hundred thousand years after the ice loading event has ceased. Hereafter we will employ the ice loading history of duration of 1 Myr, which is probably an acceptable time scale for snowball Earth formation since the time scale of the rise to the glacial maximum during each of the late Quaternary glacial cycles was ~100 kyr.

4.4. The Influence of the Shorter Duration of the Neoproterozoic Day

[56] The stabilizing effect of a shorter LOD on the rotation of the Earth is clearly seen in Figure 11b. When the length of day decreases (i.e., the spin rate increases), the TPW is reduced. For an LOD of 22 h, the polar wander is approximately 16% smaller than that obtained for an LOD of 24 h at all times in the evolution of the system. This is consistent with our qualitative estimate in section 3.1. Note that unlike the impact of ice sheet loading history, the absolute difference between the polar wander obtained for different values of the LOD increases linearly with time. At 4 Myr, the

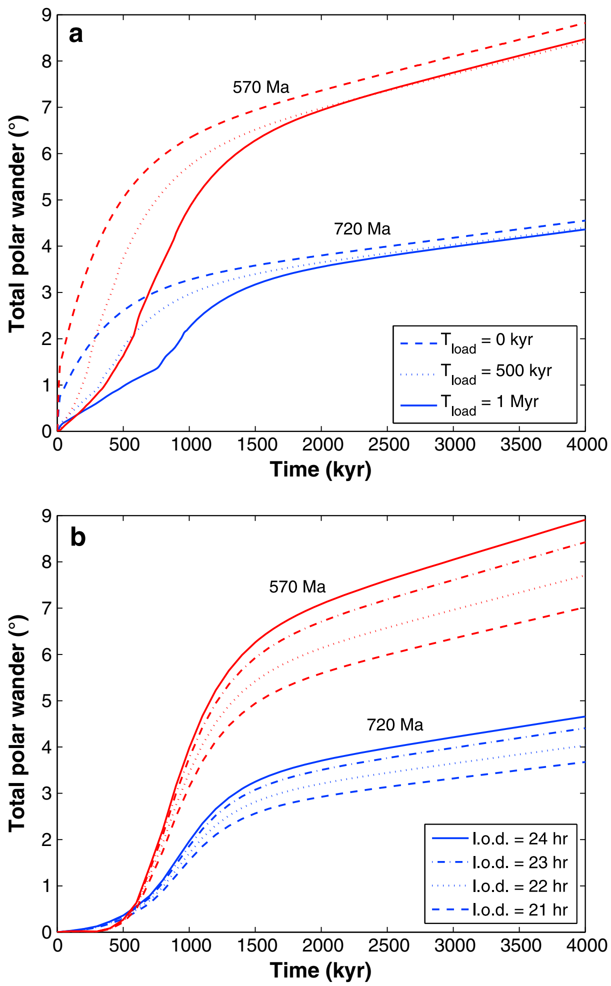

Figure 11. TPW obtained for 720 Ma (blue curves) and 570 Ma (red curves) continental configurations, respectively. (a) Ice sheets (of soft snowball extent) are emplaced on the continents instantaneously or gradually in 500 kyr and 1 Myr according to the ice history in Figure 3. (b) The length of day is varied from 21 to 24 h in steps of 1 h, and the ice loading history is 1 Myr. The viscosity model is VM5a.

difference in polar wander angles between the results for LOD values of 22 and 24 h can reach ~0.7° and ~1.2° for the 720 and 570 Ma continental configurations, respectively. Therefore, the shorter length of day has a larger influence on TPW than does the ice sheet loading history. However, its influence on relative sea level is smaller. The maximum difference in relative sea level obtained for an LOD of 22 and 24 h is smaller than 2 m at essentially all times (results not shown). This is because the TPW speeds obtained for the two LOD values remain close for all time. Nevertheless, we will fix the assumed value of the LOD to 22 h as is appropriate for the Neoproterozoic [*Schmidt and Williams*, 1995] for the purpose of all further discussions.

[57] Figure 11 also shows that the polar wander speed is much faster for the 570 Ma continental configuration than for the 720 Ma continental configuration. This is due to the combined influence of both their latitudinal positions (Figures 2 and 10) and the volume of the ice sheets (Figure 3) predicted by the ice sheet coupled climate model for the two cases.

4.5. The Influence of Excess Ellipticity as Represented by ε

[58] With the introduction of nonzero ε, the long time scale rotational stability of the planet is dramatically increased (an example for the 570 Ma continental configuration is shown

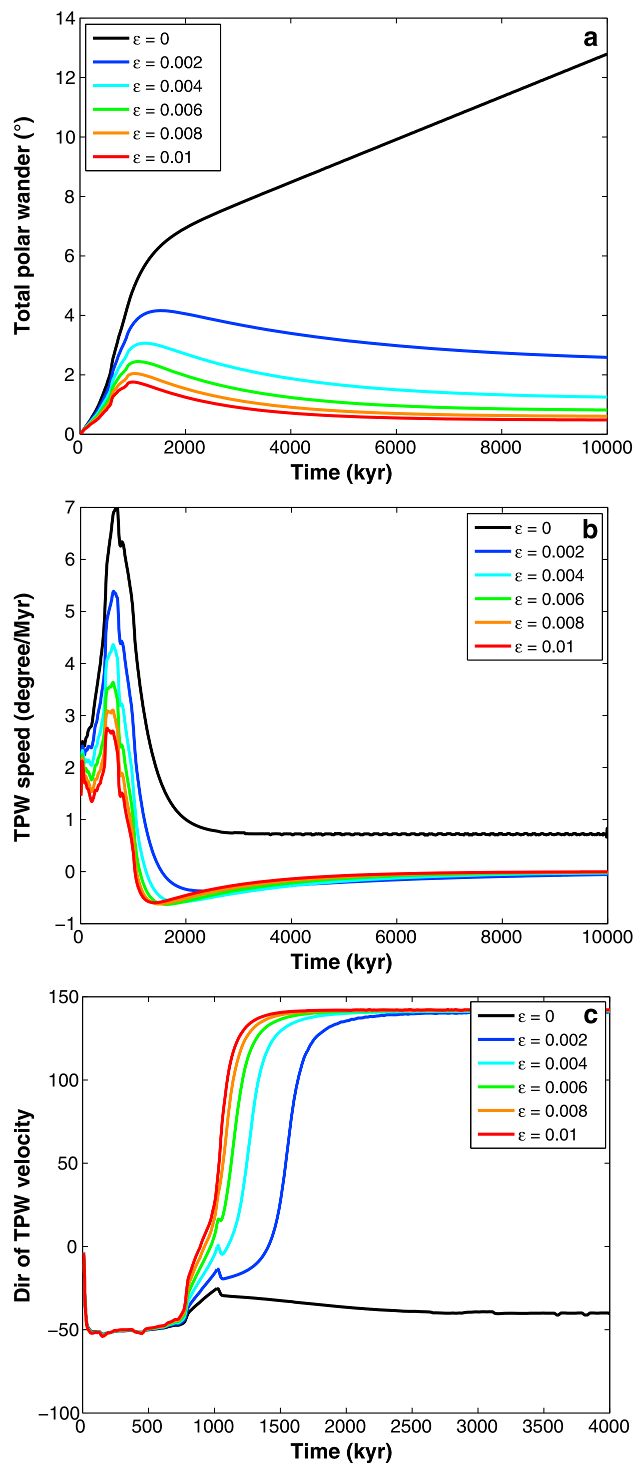

Figure 12. (a) TPW angle, (b) TPW speed, and (c) TPW direction obtained for the 570 Ma continental configuration when different magnitudes (indicated by ε) of excess ellipticity are considered. The viscosity model is VM5a, and the LOD is 22 h.

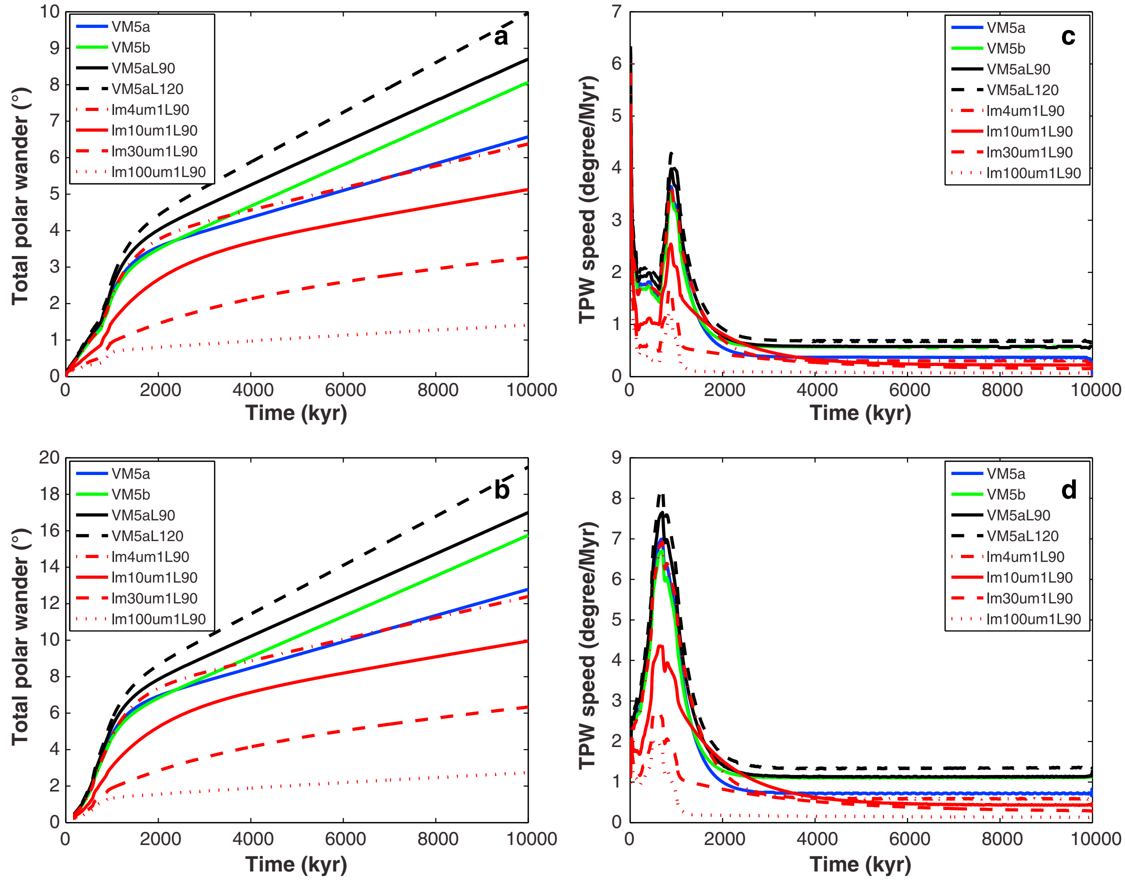

Figure 13. TPW and TPW speed obtained for (a, c) 720 Ma and (b, d) 570 Ma continental configurations, respectively, for all eight viscosity models. The LOD is 22 h, and the snowball Earth formation is completed in 1 Myr.

in Figure 12). Due to the action of ε (or the equivalent mass load at the equator it represents), the rotational axis tends to return to (although it never reaches in finite time) its original position after an initial deviation due to ice loading. For viscosity model VM5a, the rotational axis begins to return to its original position 100–600 kyr (Figure 12c) after the ice sheet loading event is completed (at $t = 1$ Myr) depending on the magnitude of ε. This is because for smaller ε, the virtual mass load at the equator is not large enough compared to the ice loading at higher latitude until the ice loading is almost compensated by isostatic adjustment of the mantle. When $\varepsilon = 0$, the rotational axis continues to "wander" away from its original position with time and moves more than 6° and 12° in 10 Myr for the 720 Ma (not shown) and 570 Ma (Figure 12a) continental configurations, respectively. However, even with a small value of ε (= 0.002), the rotational axis will return to within ~1.4° and ~2.6° of the original position for the two continental configurations, respectively, 9 Myr after the snowball formation event is completed for this particular viscosity model (VM5a). Moreover, the rotational axis is almost fully stabilized after 10 Myr because with such an angular drift due to TPW, the ice loading at higher latitude is almost balanced by the extra mass loading at the equator due to the excess ellipticity.

[59] Nonzero ε will always play a significant role in the long-term TPW of Earth for all the viscosity models tested. However, when the viscosity of the lower mantle is high (results not shown), the angular amount of TPW may continue to increase with time after 10 Myr even for ε as high as 0.004. The reason for this is that TPW increases very slowly with time (Figure 13), so that the mass loading due to excess ellipticity has not been shifted away from the equatorial plane significantly even after 10 Myr, constituting a negligible restoring force to the ongoing TPW.

[60] It is worth mentioning that the polar wander speed (Figure 12b) from our calculations is consistent with the theoretical estimate obtained by *Tsai and Stevenson* [2007] based on a scaling argument. The maximum TPW speed is obviously achieved for the 570 Ma continental configuration loaded with ice sheets when $\varepsilon = 0$. The maximum speed is ~8° Myr^{-1} for this case, comparable to the value that would be obtained for a mean mantle viscosity of ~4×10^{21} Pa s based on *Tsai and Stevenson* [2007, equation (18)], which also did not include the influence of any "excess ellipticity" in their analysis of the TPW "speed limit."

[61] Although the maximum relative sea level changes due to TPW can be as large as a few tens of meters around the time ice loading is completed, the change becomes negligible (only a few meters, results not shown) a few million years later for all $\varepsilon > 0$ cases and all viscosity models. Therefore,

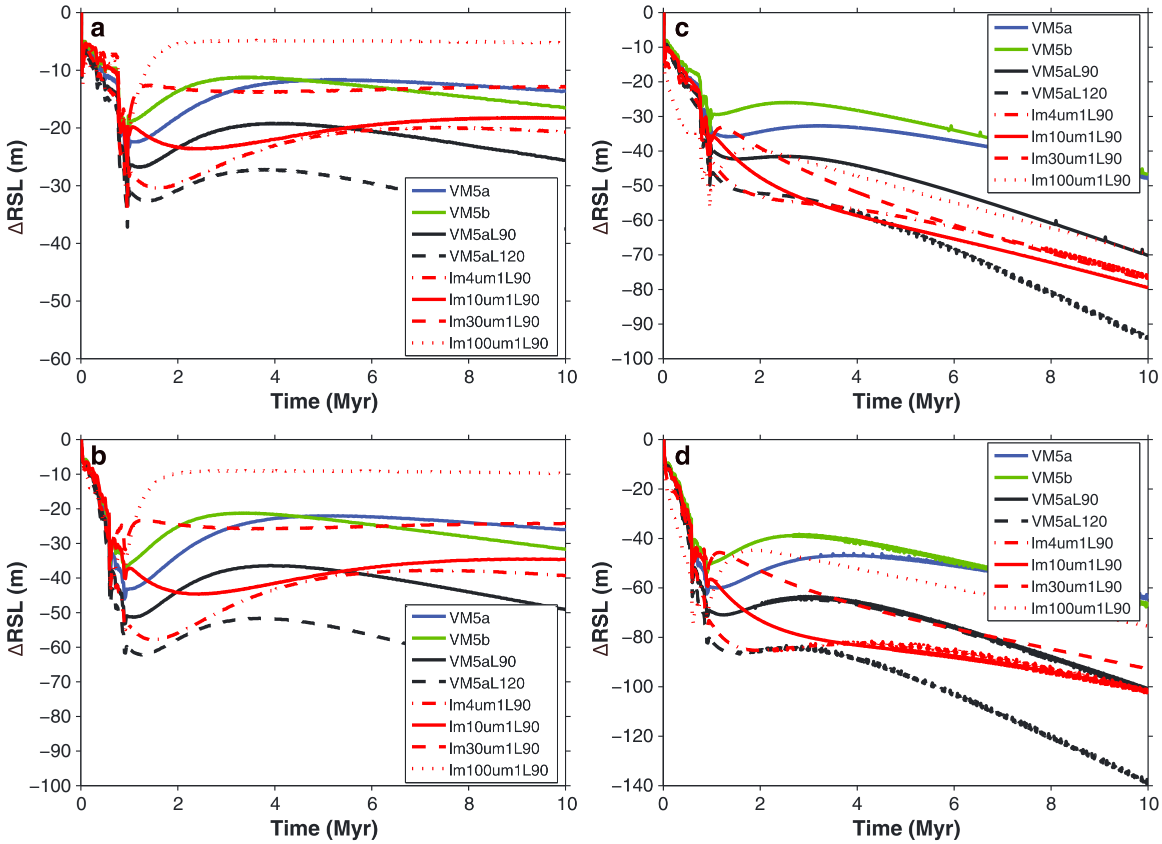

Figure 14. Time series of relative sea level change at (180°E, 30°N) due to rotational feedback only in response to the formation of snowball Earth for (a, c) 720 Ma and (b, d) 570 Ma continental configurations. The panels to the right further consider the influence of a superimposed TPW of 1°/Myr due to mantle convection. The relative sea level change shown here is obtained by subtracting the total relative sea level change calculated without considering any rotational effect (i.e., as done in *Liu and Peltier* [2013]) from that calculated in this paper.

relative sea level changes due to rotational feedback need not be considered if the Earth had nonzero excess ellipticity during the Neoproterozoic.

4.6. The Influence of the Radial Viscosity Structure

[62] The angular amount of TPW expected due to the glaciation of a supercontinent during a snowball Earth event is highly sensitive to the model assumed for the radial viscoelastic structure of the Earth (Figure 13). TPW increases with the thickness of the lithosphere but decreases with the viscosity of the lower mantle. This is easily understandable since an applied ice mass load will be less compensated by thicker lithosphere, resulting in larger forcing of TPW, while higher viscosity of the lower mantle makes it harder for the equatorial bulge to adjust to the new rotational axis, resulting in a stronger stabilizing force for the TPW. Interestingly, the TPW obtained for the VM5a model (blue solid curves in Figure 13) is in the middle of the ranges obtained for all the viscosity models and is very similar to that obtained for the lm4um1L90 model (red dash-dotted curves in Figure 13).

[63] Figure 13 also shows that the linear method employed to calculate the TPW of the Earth is entirely suitable for studying the TPW of a snowball Earth event for the 720 Ma continental configuration at least for the first 10 Myr, but it may not be suitable for some cases (of viscosity models) of the 570 Ma continental configuration, for which the method is suitable only for the first 5 Myr for almost all the viscosity models, except the one with very thick lithosphere (black dashed curve in Figure 13b). However, the results in Figure 9 demonstrate that continuing the calculations for a longer time will result in an overestimate of the relative sea level change (at the optimal location, i.e., around 30°N or 30°S along the great circle of TPW) by at most 1.5 m/Myr. This is established by the fact that the TPW curves (Figure 9a) employed to calculate the relative sea level changes (Figures 9b and 9c) represent the fastest TPW obtained for the realistic snowball Earth events in Figure 13 and the expectation that the TPW after 10 Myr will become slightly slower (since the ice loading is being continuously compensated and moving toward low latitude) than that obtained on the basis of the linear calculations. Therefore, the relative sea level change obtained from the linear calculations should represent a very good estimate for most cases (of different viscosity models and ice sheets) and provide a reasonable upper bound estimate for the other cases.

[64] Figure 14 shows the relative sea level change due to the rotational effect alone, which is obtained by subtracting the relative sea level change obtained in *Liu and Peltier* [2013] (in which the rotational influence was neglected) from that obtained in this paper for each corresponding Earth model

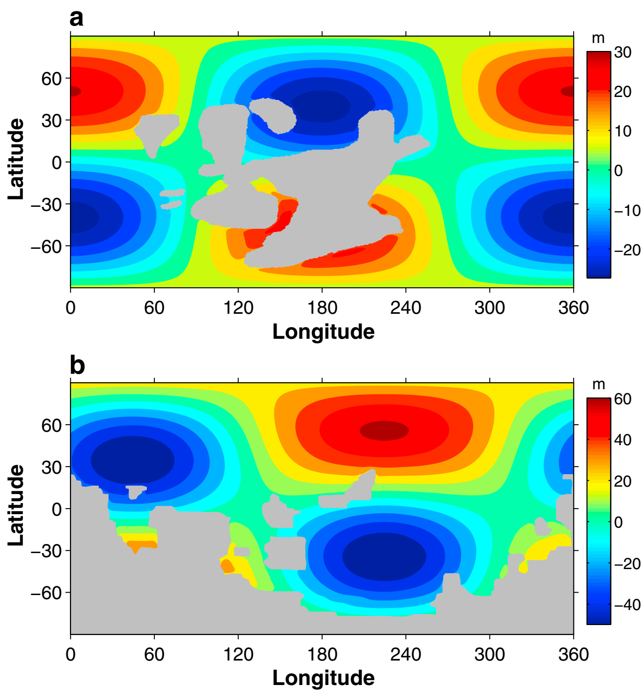

Figure 15. Relative sea level change due to rotational feedback at 10 Myr for (a) 720 Ma and (b) 570 Ma continental configurations, respectively. The LOD is 22 h, and the viscosity model is VM5aL90. Excess ellipticity is not considered.

and ice sheet loading history. Consistent with what has been demonstrated in Figure 9, the magnitude of the relative sea level change (which may be either positive or negative) will first increase quickly and then rapidly relax after the snowball Earth formation event is completed and then begin to increase slowly again after a few million years. For high lower mantle viscosity models, this feature is not nearly as obvious because the TPW speed for them is small even during the snowball formation event itself.

[65] The results in Figure 9 also demonstrate that increasing the viscosity of any part of the mantle (including increasing the thickness of the lithosphere) will enhance the relative sea level change for the same prescribed TPW. Together with the results in Figure 13, which demonstrate that increasing the thickness of the lithosphere will greatly enhance the TPW, we can expect that increasing the thickness of the elastic lithosphere will act to significantly enhance the relative sea level change (due to rotational influence alone) given the same ice sheet loading. This is clearly demonstrated in Figure 14. When the thickness of the elastic lithosphere is increased from approximately 60 km (VM5a) to 120 km (VM5aL120), the relative sea level change due to rotational influence alone is almost doubled at 3 Myr, and this difference continues to grow with time (compare the black dashed curves with the blue solid curves in Figures 14a and 14b). While the net effect of increasing lower mantle viscosity acts to monotonically reduce the relative sea level change (compare the four red curves in Figures 14a and 14b), because although it enhances the relative sea level change (for a prescribed TPW event) as depicted in Figure 9, it also acts to dramatically stabilize the rotational axis of the Earth (i.e., produces slower TPW, see Figure 13), and this latter effect dominates.

[66] Although the TPW obtained for VM5a is in the middle of the ranges (Figures 13a and 13b) obtained for all viscosity models, the relative sea level change obtained for such TPW is, however, among the smallest of all (Figures 14a and 14b), except that obtained for the very high lower mantle viscosity model (i.e., lm100um1L90, red dotted curve). The value of the relative sea level change obtained for VM5a at 10Myr is only ~14 and ~25 m for 720 Ma (blue solid curve in Figure 14a) and 570 Ma (blue solid curve in Figure 14b) continental configurations, respectively, while increasing the thickness of the elastic lithosphere from 60 km (VM5a) to only 90 km (VM5aL90) will increase the relative sea level change significantly to ~26 and ~49 m (black solid curves in Figures 14a and 14b), respectively, almost doubling the amount. However, these are still small (less than 10%) compared to the mean freeboard changes of ~400 and 600 m during the snowball Earth events for 720 and 570 Ma continental configurations [*Liu and Peltier*, 2013], respectively.

4.7. Including the Influence of TPW Due to Mantle Convection

[67] We have also tested the cases in which a 1°/Myr TPW due to mantle convection is superimposed on the TPW due to realistic snowball Earth events, and the results are shown in Figures 14b and 14d for the 720 and 570 Ma continental configurations, respectively. It turns out that this 1°/Myr TPW has a very significant influence on relative sea level change, with the influence being most significant for the high lower mantle viscosity models simply because, for these models, the ice loading–induced TPW speed was very slow. For these high lower mantle viscosity models, the relative sea level change at 10 Myr is now similar to that obtained for VM5aL90. For the low viscosity models VM5a and VM5b, the relative sea level change at 10 Myr is now ~47 and ~65 m for the 720 and 570 Ma continental configurations, respectively, about 3 and 2.6 times those obtained in the absence of this superimposed TPW. For model VM5aL90, which has a slightly thicker elastic lithosphere than VM5a, the relative sea level change is now ~70 and 101 m for the two continental configurations, respectively. These two values are no longer negligible (~16% for both continental configurations) compared to the mean freeboard changes due to snowball Earth formation. More importantly, the magnitude of the relative sea level change for VM5aL90 is increasing at a higher rate (~5 m/Myr) than all other viscosity models, except VM5aL120. Therefore, when TPW due to mantle convection is considered, the relative sea level change due to Earth's rotation becomes much more significant, and it may be even more significant if the elastic lithosphere is thicker than 90 km and snowball Earth events last longer than 10 Myr.

[68] If the duration of snowball Earth events were shorter than 10 Myr, e.g., 5 Myr, the relative sea level change associated with the changing rotation would be 35, 47, and 62 m for the 720 Ma continental configuration for viscosity models VM5a, VM5aL90, and lm10um1L90, respectively. For the 570 Ma continental configuration, these respective values are 48, 70, and 85 m. These values are smaller than those at 10 Myr but still represent more than 10% (taking the values for the viscosity model VM5aL90 as an example) of the mean freeboard values.

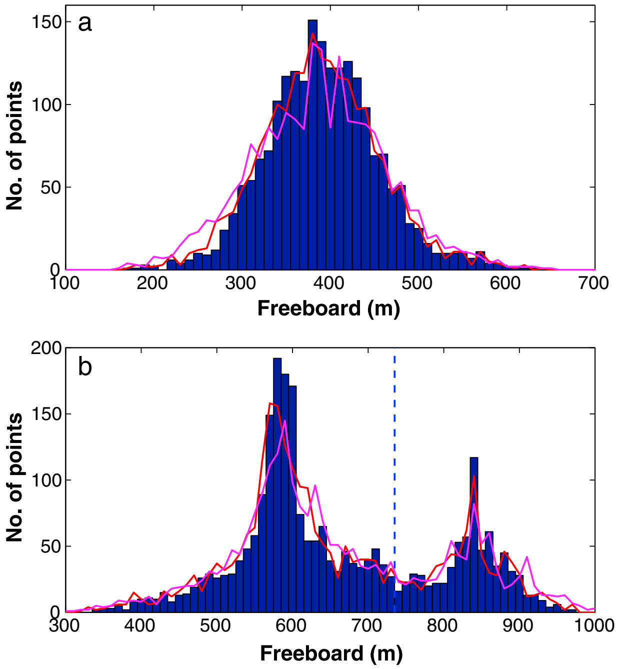

Figure 16. Histograms of freeboard at an interval of 10 m for all the coastal points, which are identified on a 0.5°×0.5° gridded map, for (a) 720 Ma and (b) 570 Ma continental configurations at 10 Myr. The viscosity model is VM5aL90. The blue histogram is that obtained when rotational feedback is not considered, while the red curves are the envelopes of new histograms in which rotational feedback is considered. The additional influence of TPW due to superimposed mantle convection is illustrated by the pink curves. The vertical blue dashed lines are the mean (over the ocean) ocean surface height lowering due to ice sheet growth.

4.8. Detailed Relative Sea Level Change During Snowball Earth Events Along the Coasts of the Glaciated Supercontinents

[69] The discussions above have focused on the relative sea level change at the optimal point at which the relative sea level change is most significant for different models of the TPW process. However, how much influence the rotational feedback would have on freeboard (relative sea level at the edge of continents), and which would actually be observable from coastal sedimentological records, clearly depends on the detailed geography of the supercontinent. Figure 15 shows the relative sea level change due to rotational feedback at 10 Myr for both the 720 and 570 Ma continental configurations. It can be seen that at this time (10 Myr following the onset of the glaciation event), the maximum relative sea level changes do affect the freeboard values along the coastline of the 720 Ma continents (Figure 15a), but not as significantly as that of the 570 Ma continent (Figure 15b). However, because the locations of peak relative sea level changes evolve with time if significant TPW occurs such as for the 570 Ma continental configuration and ice sheets, the location of one of the peaks of the relative sea level change at 5 Myr does appear around the coastline for this continental configuration (result not shown).

[70] It is useful to reiterate the idea that a perturbation to the rotation of Earth does not change the volume of ocean water but only redistributes it; therefore, we should not expect that the action of rotational feedback would change the overall freeboard values significantly even if the effect is very strong. This is verified by looking at Figure 16, which shows the histogram of freeboard values due to the integrated effect of ocean mass change (due to ice sheet formation) and to the changing Earth's rotation. After rotational feedback is considered (the red and pink curves in Figure 16), the maximum-likelihood values (the location of the peaks in the histogram) are almost the same for both the 720 and 570 Ma continental configurations relative to the histogram that does not consider this feedback (the blue histogram in Figure 16). However, the implied probability distribution (the envelope of the histogram) is broadened when the influence of rotational feedback is included. This is understandable since in the absence of rotational feedback, the relative sea level (and the ocean surface) is almost uniform [*Liu and Peltier*, 2013], while including this feedback increases the variability of relative sea level (Figure 15) and therefore increases the variability of the freeboard values.

[71] The increased width of the freeboard histogram is clearly problematic when one seeks to employ a few isolated inferences of freeboard change inferred on the basis of Neoproterozoic coastal sedimentology to infer the amount by which sea level must have fallen to build the Neoproterozoic continental ice sheets. The histogram can be treated as a probability density distribution when appropriately normalized. Distributions with less distinct peaks are very difficult to be distinguished from each other on the basis of a limited number of samples. For example, if a freeboard value of 250 m during the Sturtian snowball Earth period were observed at only two or three locations, we could confidently say that the snowball Earth hypotheses should be rejected based on the blue histogram in Figure 16a. However, due to the action of the rotational feedback, this histogram has been broadened, and we are then unable to support this conclusion as confidently because such small freeboard values are now possible even if very unlikely.

5. Conclusions

[72] We have studied in detail the influence of snowball Earth formation on the planet's rotational state as well as its feedback onto sea level. Based on our analyses involving prescribed changes of rotational state, the influence on relative sea level change due to the deceleration of Earth's rotation due to tidal friction is negligible, but that due to the TPW process can be >100 m depending on the radial viscosity structure of the Earth and the TPW speed (within the range that may be caused by a snowball Earth event). Moreover, the magnitude of relative sea level change (whether positive or negative) would be expected to increase with time even for a constant TPW speed as low as 0.5°/Myr (results not shown). The relative sea level change relaxes quickly when the TPW speed is reduced abruptly. For a fixed TPW speed, the relative sea level change increases significantly with the increase of the thickness of the elastic lithosphere, but much less significantly with an increase of the lower mantle viscosity unless the TPW speed is large (e.g., the results presented for a TPW speed of 8°/Myr, Figure 6b). This agrees, in general, with the conclusion of *Mound et al.* [1999], although they did not investigate the influence of such high polar wander speeds. Furthermore,

these experiments with prescribed TPW demonstrate that the relative sea level change obtained by linear perturbation theory for a detailed model involving an explicitly described snowball Earth continental glaciation event may be overestimated when the angular amount of TPW is >8.1° (i.e., when the assumption of linear perturbation theory is invalid). This overestimation will grow with time, but by only ~1.5 m/Myr (the exact value depending on the viscosity model employed), and therefore is not expected to be important.

[73] From the calculations based on the self-consistent use of linear perturbation theory, it is found that different durations of the snowball Earth formation event have a negligible influence on both TPW and relative sea level change if we are interested only in the results >1.2 Myr after the onset of the glaciation event. The variation of the LOD has a relatively larger influence on the TPW of the Earth but a negligible influence on relative sea level change. Excess ellipticity has a very significant influence on both TPW and relative sea level change. TPW speed is reduced almost to 0 after a few million years even for a very small value of ε, e.g., 0.002, and the relative sea level change associated with the TPW will therefore be negligible. Although we are able to estimate the excess ellipticity of the planet under modern conditions, it remains unclear whether any such influence may have been active under Neoproterozoic ice age conditions, during which tectonic processes were highly active due to the breakup of the supercontinent of Rodinia.

[74] For the same ice loading due to snowball Earth formation, the TPW induced will be significantly enhanced for larger values of the thickness of the elastic lithosphere. Since the relative sea level change will also be enhanced for a fixed TPW event when the thickness of the elastic lithosphere is increased, the relative sea level change during a snowball Earth period is therefore most sensitive to the thickness of the elastic lithosphere. In contrast, increasing the viscosity of the lower mantle will reduce TPW and relative sea level change dramatically. When both the viscosity of the lower mantle and the thickness of the lithosphere are increased, for example, by changing the viscosity profile from VM5a to lm30um1L90 for which lower mantle viscosity is increased by almost an order of magnitude and the thickness of the lithosphere only by a factor of 1.5 from ~60 to 90 km, the TPW will be dramatically reduced (compare the red dashed curves to the blue solid curves in Figures 13a and 13b), but the relative sea level changes are still similar (compare the red dashed curves to the blue solid curves in Figures 14a and 14b).

[75] The TPW speed obtained for the 570 Ma continental configuration and ice sheets is approximately twice that obtained for the 720 Ma continental configuration. For this reason, the perturbation theory–based method is applicable for the purpose of calculating the rotational feedback on sea level change for the latter model but not for the former for certain viscosity models (especially those with thicker lithosphere). The maximum TPW speed for the 570 Ma continental configuration can be as large as 8.0°/Myr for viscosity model VM5aL120 and zero excess ellipticity. The maximum relative sea level change at 10 Myr is only 14 and 25 m for 720 and 570 Ma continental configurations, respectively, for viscosity model VM5a, but it is significantly increased to 26 and 49 m for viscosity model VM5aL90, for which the only change is that the thickness of the elastic lithosphere is increased from 60 to 90 km. However, if mantle convection is driving a slow TPW of 1°/Myr, in the same direction as that driven by the ice loading, the maximum relative sea level change at 10 Myr for viscosity model VM5aL90 will be further increased to 70 and 101 m for the two continental configurations, respectively.

[76] Although rotational feedback may induce significant relative sea level change at some locations, it does not affect significantly the mean freeboard along the coastline of the continents as long as the continents extend across more than one quadrant of Earth's surface. If we sample the freeboard values (initially 10 m) along all the coastlines of the continents sampled at a resolutions of ~0.5°, a histogram plot of these values may be taken to represent the probability density distribution of freeboard values during a snowball Earth period. It is found that the influence of rotational variation is such that the probability distribution (the envelope of the histogram) is broadened, and the peaks become less distinct, but the locations of the extrema remain almost unchanged (Figure 16) compared to the results obtained when rotational influence is ignored. Therefore, the most likely freeboard values (at the peak of the histogram plots) during a snowball Earth period based on these theoretical calculations remain consistent with the sole observation at Namibia (~500 m) by *Hoffman et al.* [2007]. However, because of the broadening of the histogram, the most important influence of the action of rotational feedback on relative sea level change is to make the inference of actual ice volume (which could be employed to deny or establish the occurrence of a snowball Earth glaciation) during the hypothesized snowball Earth period from observed freeboard change (from sedimentological records) more susceptible to error. This must be kept in mind in the assessment of the meaningfulness of the single data point that is currently available based on analyses of the Namibia section.

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